

NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics
MA 1505 Mathematics I
Tutorial 9

1. Let $\mathbf{F}(x, y, z) = 2xy\mathbf{i} + (x^2 + 2yz)\mathbf{j} + y^2\mathbf{k}$. Show that \mathbf{F} is a conservative vector field. Find a function f such that $\nabla f = \mathbf{F}$.

Ans: $f(x, y, z) = x^2y + y^2z + K$

2. Evaluate $\int_C g(x, y, z) ds$, where $g(x, y, z) = x^2 - yz + z^2$ and C is the line segment from $(0, 0, 0)$ to $(1, 2, 3)$.

Ans: $4\sqrt{14}/3$

3. Compute the work done by the force $\mathbf{F}(x, y, z) = yz\mathbf{i} + 2y\mathbf{j} - x^2\mathbf{k}$ on a particle that moves along the curve C given by the vector function $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$, for $0 \leq t \leq 1$.

Ans: $17/30$

4. Evaluate $\int_C 2xy dx + (x^2 + z) dy + y dz$, where C consists of two line segments: C_1 from $(0, 0, 0)$ to $(1, 0, 2)$, and C_2 from $(1, 0, 2)$ to $(3, 4, 1)$.

Ans: 40

5. The base of a circular fence with radius 10 m is given by $x = 10 \cos t$, $y = 10 \sin t$. The height of the fence varies from 3 m to 5 m such that, at position (x, y) , the height is given by the function $h(x, y) = 4 + 0.01(x^2 - y^2)$. Suppose that 1 litre of paint covers 100 m^2 . Determine how much paint you will need if you paint both sides of the fence.

Ans: 5 litre

6. For each non-zero constant a , let C_a denote the curve $y = a \sin x$, $0 \leq x \leq \pi$.

Let $I(a) = \int_{C_a} (1 + y^3) dx + (2x + y) dy$.

Find the minimum value of $I(a)$ in the domain $a > 0$.