

NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics
MA 1505 Mathematics I
Tutorial 8

1. Find the volume of the solid whose base is the region in the xy -plane that is bounded by the parabola $y = 4 - x^2$ and the line $y = 3x$, while the top of the solid is bounded by the plane $x - z + 4 = 0$.

Ans: $625/12$

2. Find the volume of the solid bounded by the cylinders $x^2 + y^2 = r^2$ and $y^2 + z^2 = r^2$.

Ans: $\frac{16}{3}r^3$

3. Find the area of the surface consisting of the part of the sphere of radius 2 centered at origin that lies above the horizontal plane $z = 1$. (Equation of this sphere is given by $x^2 + y^2 + z^2 = 2^2$.)

Ans: 4π

4. Find the exact value of the surface area of the portion of the upper cone

$$z = \sqrt{x^2 + y^2}$$

above the region $D = \{(x, y) \in R^2 : x^2 \leq y \leq x + 2, -1 \leq x \leq 2\}$.

Ans: $\frac{9\sqrt{2}}{2}$.

5. Evaluate the following triple integral:

$$\iiint_D (x^2 + 2z) dV, \quad D \text{ is the solid cube } \{-\frac{1}{2} \leq x \leq \frac{1}{2}, -\frac{1}{2} \leq y \leq \frac{1}{2}, -\frac{1}{2} \leq z \leq \frac{1}{2}\}.$$

Ans: $\frac{1}{12}$

6. Find the area cut from the saddle surface $az = x^2 - y^2$ by the cylinder $x^2 + y^2 = a^2$. Here a is a positive constant.