# NATIONAL UNIVERSITY OF SINGAPORE
## Department of Mathematics

### MA 1505 Mathematics I
### Tutorial 7

1. Calculate the following iterated integrals:
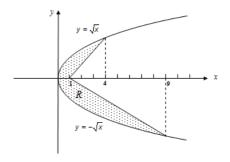
   (a) $\displaystyle\int_0^b \int_0^a (x^2 + y^2)\, dx\, dy$
   (b) $\displaystyle\int_1^2 \int_0^1 \frac{xy}{\sqrt{4 - x^2}}\, dx\, dy.$

   **Ans**: (a) $\frac{1}{3}ab(a^2 + b^2)$    (b) $3 - 3\sqrt{3}/2$

2. Evaluate the following double integrals:

   (a) $\displaystyle\iint_R e^{x^2}\, dA,$    $R$ is the region bounded by $y = 0,\ \ y = x,\ \ x = 1.$

   (b) $\displaystyle\iint_R (x + y)\, dA,$    $R$ is the region bounded by the two curves $y = \sqrt{x},\ \ y = x^2.$

   **Ans**: (a) $\frac{1}{2}(e - 1)$   (b) $\frac{3}{10}$

3. Evaluate the double integral $\iint_R x\, dA$ where $R$ is the region as shown below.



   **Ans**: 25

4. Evaluate the integral $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2}\, dy\, dx$ by converting it to polar coordinates.

   **Ans**: $\frac{1}{4}\pi(e - 1)$

5. Evaluate the following integrals by reversing the order of integration.

   (a) $\displaystyle\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\, dx\, dy$   (b) $\displaystyle\int_0^{2\sqrt{\ln 3}} \int_{y/2}^{\sqrt{\ln 3}} e^{x^2}\, dx\, dy$

   **Ans**: (a) $\frac{1}{4}(e^{16} - 1)$   (b) 2

6. Find the exact value of the double integral

$$\int\int_D \sqrt{4 - x^2 - y^2}\,dxdy$$

Where $D$ is the part of the plane in the second quadrant bounded between the two circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

**Ans:** $\frac{1}{2}\sqrt{3}\pi$.

7. Find

$\lim\limits_{t \to 0^+} \frac{1}{t^4} \int_0^t \int_x^t \sin y^2\,dydx.$