NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

$\mathbb{MA}$ 1505 Mathematics I
Tutorial 4

1. Write the vector $3\mathbf{i}+2\mathbf{j}+\mathbf{k}$ as a sum of two vectors $\mathbf{u}+\mathbf{v}$ such that $\mathbf{u}$ is parallel to $\mathbf{w} = \mathbf{i}+3\mathbf{j}+4\mathbf{k}$ and $\mathbf{v}$ is perpendicular to $\mathbf{w}$. (Hint: Use projection)

   Ans. $\frac{1}{2}(\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}) + \frac{1}{2}(5\mathbf{i} + \mathbf{j} - 2\mathbf{k})$

2. Consider the two lines:

$$\ell_1 : x = 2 + 2t, \quad y = 2 + t, \quad z = 3 + 3t$$

$$\ell_2 : x = -12 + 4t, \quad y = -5 + 2t, \quad z = -3 + t$$

   Show that ℓ_1 and ℓ_2 intersect. Find the point of intersection and an equation of the plane containing ℓ_1 and ℓ_2.

   Ans. $(0, 1, 0), -x + 2y = 2$

3. (i) Find an equation of the plane Π passing through the points $A(3, 3, 0), B(3, 0, 1)$ and $C(0, 2, 1)$.

   (ii) Find the distance of Π from $O(0, 0, 0)$.

   (iii) Let $D = (4, 2, 1)$. Find the coordinates of the point of intersection of the plane Π in part (i) and the line segment OD.

   Ans. (i) $2x + 3y + 9z = 15$; (ii) $15/\sqrt{94}$; (iii) $\frac{15}{23}(4, 2, 1)$

4. Find the shortest distance between the two planes:

$$\Pi_1 : \quad 2x + 2y - z = 1 \qquad \text{and} \qquad \Pi_2 : \quad 4x + 4y - 2z = 5.$$

   Ans. 1/2

5. Two particles travel along the space curves which, at time t, are given by:

$$\mathbf{r}_1(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}, \quad \mathbf{r}_2(t) = (1 + 2t)\mathbf{i} + (1 + 6t)\mathbf{j} + (1 + 14t)\mathbf{k}.$$

   Do the particles collide? Do their paths intersect?

6. $\mathbf{A}$ and $\mathbf{B}$ are two non-zero constant vectors and $||\mathbf{B}|| = 1$.
   If the angle between them is equal to $\frac{\pi}{4}$, find the value of $\lim_{x\to 0} \frac{||\mathbf{A}+x\mathbf{B}||-||\mathbf{A}||}{x}$.

   Ans. $\frac{\sqrt{2}}{2}$

7. It is known that the earth orbits around the sun in an elliptical orbit with the sun at one focus. Using Newton's Second Law of Motion (i.e. force = mass x acceleration) and the fact that the gravitational force exerted by the sun on the earth is a vector parallel to the vector that joins the sun to the earth, prove Kepler's Second Law of Planetary Motion: The segment from the centre of the sun to the centre of the earth sweeps out area at a constant rate. (In this problem, we assume that the gravitational pulls exerted on earth by planets other than the sun are negligible when compare to that of the sun.)