NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

MA 1505 Mathematics I
Tutorial 4

1. Find the radius of convergence of the following series.

   (a) $\sum_{1}^{\infty} (-1)^n \frac{(x+2)^n}{n}$ (b) $\sum_{1}^{\infty} \frac{(3x-2)^n}{n}$ (c) $\sum_{1}^{\infty} (-1)^n (4x+1)^n$

   (d) $\sum_{0}^{\infty} \frac{3^n x^n}{n!}$ (e) $\sum_{1}^{\infty} n^n x^n$ (f) $\sum_{1}^{\infty} \frac{(4x-5)^{2n+1}}{n^{3/2}}$

   Ans. (a) 1 (b) 1/3 (c) 1/4 (d)∞ (e) 0 (f) 1/4

2. Find the sum of the geometric series inside the interval of convergence

$$1 - \frac{1}{2}(x-3) + \frac{1}{4}(x-3)^2 - + \cdots + (-\frac{x-3}{2})^n + \cdots .$$

   Ans. $\frac{2}{x-1}$

3. Find the Taylor series for the following functions:

   (a) $\frac{x}{1-x}$ at $x = 0$;

   (b) $\frac{1}{x^2}$ at $x = 1$;

   (c) $\frac{x}{1+x}$ at $x = -2$;

   Ans. (a) $\sum_{n=0}^{\infty} x^{n+1}$ (b) $\sum_{n=0}^{\infty} (-1)^n (n+1)(x-1)^n$ (c) $2 + \sum_{n=1}^{\infty} (x+2)^n$

4. Use the Taylor polynomial $P_3(x)$ of $f(x) = \sin x$ at $x = 0$ of order 3 to approximate $\sin 0.1$. Show that the error incurred in the approximation is less than 10^{-5}.

   Ans. 0.09983

5. Let
$$S = \sum_{n=0}^{\infty} \frac{1}{n!\,(n+2)}.$$
In this question, we will introduce two different ways to find the value of S, one by integration and the other by differentiation.

(i) Integrate the Taylor series of xe^x to show that $S = 1$.

(ii) Differentiate the Taylor series of $\frac{e^x-1}{x}$ to show that $S = 1$.

6. Let n be a positive integer. Prove that
$\frac{1}{2}\int_0^1 t^{n-1}\,(1-t)^2\,dt = \frac{1}{n(n+1)(n+2)}$.

Hence find the exact value of the infinite series
$\frac{1}{1\cdot2\cdot3} + \frac{1}{3\cdot4\cdot5} + \frac{1}{5\cdot6\cdot7} + \frac{1}{7\cdot8\cdot9} + \cdots$