NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

MA 1505 Mathematics I
Tutorial 3

1. Write the vector $3\mathbf{i}+2\mathbf{j}+\mathbf{k}$ as a sum of two vectors $\mathbf{u}+\mathbf{v}$ such that $\mathbf{u}$ is parallel to $\mathbf{w} = \mathbf{i}+3\mathbf{j}+4\mathbf{k}$ and $\mathbf{v}$ is perpendicular to $\mathbf{w}$. (Hint: Use projection)

   Ans. $\frac{1}{2}(\mathbf{i}+3\mathbf{j}+4\mathbf{k}) + \frac{1}{2}(5\mathbf{i}+\mathbf{j}-2\mathbf{k})$

2. Consider the two lines:

$$\ell_1 : x = 2+2t, \quad y = 2+t, \quad z = 3+3t$$

$$\ell_2 : x = -12+4t, \quad y = -5+2t, \quad z = -3+t$$

   Show that ℓ_1 and ℓ_2 intersect. Find the point of intersection and an equation of the plane containing ℓ_1 and ℓ_2.

   Ans. $(0,1,0), -x+2y = 2$

3. (i) Find an equation of the plane Π passing through the points $A(3,3,0), B(3,0,1)$ and $C(0,2,1)$.

   (ii) Find the distance of Π from $O(0,0,0)$.

   (iii) Let $D = (4,2,1)$. Find the coordinates of the point of intersection of the plane Π in part (i) and the line segment OD.

   Ans. (i) $2x+3y+9z = 15$; (ii) $15/\sqrt{94}$; (iii) $\frac{15}{23}(4,2,1)$

4. Find the shortest distance between the two planes:

$$\Pi_1 : \quad 2x+2y-z = 1 \qquad \text{and} \qquad \Pi_2 : \quad 4x+4y-2z = 5.$$

   Ans. $1/2$

5. Two particles travel along the space curves which, at time t, are given by:

$$\mathbf{r}_1(t) = t\mathbf{i}+t^2\mathbf{j}+t^3\mathbf{k}, \quad \mathbf{r}_2(t) = (1+2t)\mathbf{i}+(1+6t)\mathbf{j}+(1+14t)\mathbf{k}.$$

   Do the particles collide? Do their paths intersect?

6. $\mathbf{A}$ and $\mathbf{B}$ are two non-zero constant vectors and $||B|| = 1$. If the angle between them is equal to $\frac{\pi}{4}$, find the value of $\lim_{x\to 0} \frac{||\mathbf{A}+x\mathbf{B}||-||\mathbf{A}||}{x}$

   Ans. $\frac{\sqrt{2}}{2}$