

NATIONAL UNIVERSITY OF SINGAPORE  
Department of Mathematics  
MA 1505 Mathematics I  
Tutorial 2

1. Use L'Hopital's rule to find the following limits.

$$\begin{array}{ll} \text{(a)} \lim_{x \rightarrow \pi/2} \frac{1 - \sin x}{1 + \cos 2x} & \text{(b)} \lim_{x \rightarrow 0} \frac{\ln(\cos ax)}{\ln(\cos bx)}, \quad a, b > 0 \\ \text{(c)} \lim_{x \rightarrow \infty} x \tan \frac{1}{x} & \text{(d)} \lim_{x \rightarrow 0^+} x^a \ln x, \quad a > 0 \\ \text{(e)} \lim_{x \rightarrow 1} x^{\frac{1}{1-x}} & \text{(f)} \lim_{x \rightarrow 0^+} x^{\sin x} \\ \text{(g)} \lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right)^{\frac{1}{x^2}} \end{array}$$

**Ans.** (a)  $\frac{1}{4}$  (b)  $\frac{a^2}{b^2}$  (c) 1 (d) 0 (e)  $e^{-1}$  (f) 1 (g)  $e^{-1/6}$

2. Evaluate the following definite integrals.

$$\begin{array}{ll} \text{(a)} \int_1^{\sqrt{2}} \frac{s^2 + \sqrt{s}}{s^2} ds. & \\ \text{(b)} \int_{-4}^4 |x| dx. & \\ \text{(c)} \int_0^\pi \frac{1}{2} (\cos x + |\cos x|) dx. & \\ \text{(d)} \int_0^\pi \sin^2 \left( 1 + \frac{\theta}{2} \right) d\theta. & \end{array}$$

**Ans.** (a)  $1 + \sqrt{2} - 2^{3/4}$  (b) 16 (c) 1 (d)  $\frac{1}{2}\pi + \sin 2$

3. Using the fundamental theorem of Calculus, find the derivative  $dy/dx$  for the following functions.

$$\begin{array}{ll} \text{(a)} y = \int_0^{\sqrt{x}} \cos t dt. & \\ \text{(b)} y = \int_0^{x^2} \cos \sqrt{t} dt. & \\ \text{(c)} y = \int_0^{\sin x} \frac{dt}{\sqrt{1-t^2}}, \quad |x| < \frac{\pi}{2}. & \end{array}$$

**Ans.** (a)  $\frac{\cos \sqrt{x}}{2\sqrt{x}}$  (b)  $2x \cos x$  (c) 1

4. Using the *substitution* method, or otherwise, find the following integrals.

(a)  $\int x^{1/2} \sin(x^{3/2} + 1) dx.$

(b)  $\int \csc^2 2t \cot 2t dt.$

(c)  $\int \frac{1}{\theta^2} \sin \frac{1}{\theta} \cos \frac{1}{\theta} d\theta.$

(d)  $\int \frac{18 \tan^2 x \sec^2 x}{(2 + \tan^3 x)} dx.$

(e)  $\int \frac{\sin \sqrt{\theta}}{\sqrt{\theta} \cos^3 \sqrt{\theta}} d\theta.$

**Ans.**

(a)  $-\frac{2}{3} \cos(x^{3/2} + 1) + C$

(b)  $-\frac{1}{4} \cot^2 2t + C$

(c)  $-\frac{1}{2} \sin^2 \frac{1}{\theta} + C$

(d)  $6 \ln |\tan^3 x + 2| + C$

(e)  $\sec^2 \sqrt{\theta} + C$

5. Applying the method of *integration by parts*, or otherwise, find the following integrals.

(a)  $\int x \sin \left( \frac{x}{2} \right) dx.$

(b)  $\int t^2 e^{4t} dt.$

(c)  $\int e^{-y} \cos y dy.$

(d)  $\int \theta^2 \sin(2\theta) d\theta.$

(e)  $\int z (\ln z)^2 dz.$

(f)  $\int \{\sin e^{-x} + e^x \cos e^{-x}\} dx.$

**Ans.**

(a)  $-2 \left[ x \cos \left( \frac{x}{2} \right) - 2 \sin \left( \frac{x}{2} \right) \right] + C$

(b)  $\left( \frac{t^2}{4} - \frac{t}{8} + \frac{1}{32} \right) e^{4t} + C$

(c)  $\frac{e^{-y}}{2} (\sin y - \cos y) + C$

(d)  $-\frac{1}{2} \left[ \theta^2 \cos(2\theta) - \theta \sin(2\theta) - \frac{1}{2} \cos(2\theta) \right] + C$

(e)  $\frac{1}{2} \left[ z^2 (\ln z)^2 - z^2 (\ln z) + \frac{z^2}{2} \right] + C$

(f)  $e^x \cos e^{-x} + C$

6. Find the area of the following region.

- (a) The region bounded between  $y = \frac{1}{2} \sec^2 x$ ,  $y = -4 \sin^2 x$ ,  $x = -\frac{\pi}{3}$  and  $x = \frac{\pi}{3}$ .
- (b) The region in the first quadrant bounded by  $y = x$ ,  $y = \frac{1}{4}x^2$  and below  $y = 1$ .
- (c) The region between the graphs of  $y = 4 - x^2$  and  $y = 2 - x$  from  $x = -2$  to  $x = 3$ .

**Ans.** (a)  $\frac{4}{3}\pi$  (b)  $\frac{5}{6}$  (c)  $\frac{49}{6}$

7. Find the exact value of  $\int_0^a \frac{dx}{x + \sqrt{a^2 - x^2}}$ , where  $a$  is a positive constant.