NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

MA 1505 Mathematics I
Tutorial 2

1. Use L'Hopital's rule to find the following limits.

   (a) $\lim\limits_{x\to\pi/2} \dfrac{1-\sin x}{1+\cos 2x}$ (b) $\lim\limits_{x\to 0} \dfrac{\ln(\cos ax)}{\ln(\cos bx)}, \quad a,\ b>0$

   (c) $\lim\limits_{x\to\infty} x\tan\dfrac{1}{x}$ (d) $\lim\limits_{x\to 0+} x^a \ln x, \quad a>0$

   (e) $\lim\limits_{x\to 1} x^{\frac{1}{1-x}}$ (f) $\lim\limits_{x\to 0^+} x^{\sin x}$

   (g) $\lim\limits_{x\to 0} \left(\dfrac{\sin x}{x}\right)^{\frac{1}{x^2}}$

   Ans. (a) $\frac{1}{4}$ (b) $\frac{a^2}{b^2}$ (c) 1 (d) 0 (e) e^{-1} (f) 1 (g) $e^{-1/6}$

2. Evaluate the following definite integrals.

   (a) $\displaystyle\int_1^{\sqrt{2}} \frac{s^2+\sqrt{s}}{s^2}\,ds\,.$

   (b) $\displaystyle\int_{-4}^{4} |x|\,dx\,.$

   (c) $\displaystyle\int_0^{\pi} \frac{1}{2}(\cos x + |\cos x|)\,dx\,.$

   (d) $\displaystyle\int_0^{\pi} \sin^2\left(1+\frac{\theta}{2}\right)d\theta\,.$

   Ans. (a) $1+\sqrt{2}-2^{3/4}$ (b) 16 (c) 1 (d) $\frac{1}{2}\pi+\sin 2$

3. Using the fundamental theorem of Calculus, find the derivative dy/dx for the following functions.

   (a) $y=\displaystyle\int_0^{\sqrt{x}} \cos t\,dt\,.$

   (b) $y=\displaystyle\int_0^{x^2} \cos\sqrt{t}\,dt\,.$

   (c) $y=\displaystyle\int_0^{\sin x} \frac{dt}{\sqrt{1-t^2}}, \qquad |x|<\frac{\pi}{2}\,.$

   Ans. (a) $\frac{\cos\sqrt{x}}{2\sqrt{x}}$ (b) $2x\cos x$ (c) 1

4. Using the *substitution* method, or otherwise, find the following integrals.

   (a) $\displaystyle\int x^{1/2}\sin(x^{3/2}+1)\,dx\,.$

   (b) $\displaystyle\int \csc^2 2t\,\cot 2t\,dt\,.$

   (c) $\displaystyle\int \frac{1}{\theta^2}\sin\frac{1}{\theta}\cos\frac{1}{\theta}\,d\theta\,.$

   (d) $\displaystyle\int \frac{18\tan^2 x\sec^2 x}{(2+\tan^3 x)}\,dx\,.$

   (e) $\displaystyle\int \frac{\sin\sqrt{\theta}}{\sqrt{\theta}\cos^3\sqrt{\theta}}\,d\theta\,.$

   Ans.
   (a) $-\frac{2}{3}\cos(x^{3/2}+1)+C$
   (b) $-\frac{1}{4}\cot^2 2t+C$
   (c) $-\frac{1}{2}\sin^2\frac{1}{\theta}+C$
   (d) $6\,\ln|\tan^3 x+2|+C$
   (e) $\sec^2\sqrt{\theta}+C$

5. Applying the method of *integration by parts*, or otherwise, find the following integrals.

   (a) $\displaystyle\int x\,\sin\left(\frac{x}{2}\right)\,dx\,.$

   (b) $\displaystyle\int t^2\,e^{4t}\,dt\,.$

   (c) $\displaystyle\int e^{-y}\,\cos y\,dy\,.$

   (d) $\displaystyle\int \theta^2\,\sin(2\theta)\,d\theta\,.$

   (e) $\displaystyle\int z\,(\ln z)^2\,dz\,.$

   (f) $\displaystyle\int\{\sin e^{-x}+e^x\cos e^{-x}\}\,dx\,.$

   Ans.
   (a) $-2\,\left[x\cos\left(\frac{x}{2}\right)-2\sin\left(\frac{x}{2}\right)\right]+C$
   (b) $(\frac{t^2}{4}-\frac{t}{8}+\frac{1}{32})e^{4t}+C$
   (c) $\frac{e^{-y}}{2}(\sin y-\cos y)+C$
   (d) $-\frac{1}{2}\,\left[\theta^2\cos(2\theta)-\theta\sin(2\theta)-\frac{1}{2}\cos(2\theta)\right]+C$
   (e) $\frac{1}{2}\left[z^2(\ln z)^2-z^2(\ln z)+\frac{z^2}{2}\right]+C$
   (f) $e^x\cos e^{-x}+C$

6. Find the area of the following region.

   (a) The region bounded between $y = \frac{1}{2}\sec^2 x$, $y = -4\sin^2 x$, $x = -\frac{\pi}{3}$ and $x = \frac{\pi}{3}$.

   (b) The region in the first quadrant bounded by $y = x$, $y = \frac{1}{4}x^2$ and below $y = 1$.

   (c) The region between the graphs of $y = 4 - x^2$ and $y = 2 - x$ from $x = -2$ to $x = 3$.

   Ans. (a) $\frac{4}{3}\pi$ (b) $\frac{5}{6}$ (c) $\frac{49}{6}$

7. Find $\int \frac{1}{x^7+x} dx$, where $x > 0$.