NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

$\mathbb{MA}$ 1505 Mathematics I

Tutorial 11

1. Evaluate $\iint_S f(x,y,z)\,dS$ and $\iint_S \mathbf{F} \bullet dS$, where $f(x,y,z) = x+y+z$ and $\mathbf{F} = x^2\mathbf{i}+y^2\mathbf{j}+z^2\mathbf{k}$ and S is the surface defined parametrically by

$$\mathbf{r}(u,v) = (2u+v)\mathbf{i} + (u-2v)\mathbf{j} + (u+3v)\mathbf{k}, \quad (0 \le u \le 1,\ 0 \le v \le 2).$$

   The orientation of S is given by the normal vector $\mathbf{r}_u \times \mathbf{r}_v$.

   Ans: $40\sqrt{3}$; $\quad -\frac{430}{3}$

2. Evaluate $\iint_S z\ dS$, where S is the portion of the paraboloid $z = 4 - x^2 - y^2$ lying on and above the xy plane.

   Ans: $\frac{289}{60}\pi\sqrt{17} - \frac{41}{60}\pi$

3. Evaluate $\iint_S \mathbf{F} \bullet dS$, where $\mathbf{F} = y\mathbf{i}+x^2\mathbf{j}+z^2\mathbf{k}$ and S is the portion of the plane $3x+2y+z=6$ in the first octant.

   The orientation of S is given by the upward normal vector.

   Ans: 31

4. Use Stoke's Theorem to evaluate $\oint_C \left(\frac{1}{2}y^2\ dx + z\ dy + x\ dz\right)$, where C is the curve of intersection of the plane $x+z=0$ and the ellipsoid $x^2+2y^2+z^2=1$, oriented counterclockwise as seen from above.

   Ans: $-\frac{\pi}{2}$

5. Use Stoke's Theorem to evaluate $\iint_S (\text{curl } F) \bullet d\mathbf{S}$, where $\mathbf{F}(x,y,z) = y\mathbf{i} - x\mathbf{j} + yz\mathbf{k}$ and S is part of the surface $z = 2(x^2+y^2)$ for which $z \le 1/2$.

   The orientation of S is given by the outer normal vector.

   Ans: $\frac{\pi}{2}$

6. Use the divergence theorem to evaluate $\iint_S \mathbf{F} \bullet d\mathbf{S}$, where $\mathbf{F}(x,y,z) = x^2\mathbf{i} + xy\mathbf{j} + x^3y^3\mathbf{k}$ and S is the surface of the rectangular region bounded by the three coordinate planes and the planes $x=1,\ y=2,\ z=-3$.

   The orientation of S is given by the outer normal vector.

   Ans: 9

7. Let S denote a plane with unit normal vector $\mathbf{n} = a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$. C is a simple closed piecewise-smooth curve on S and the orientation of C is positive with respect to $\mathbf{n}$. Let D denote the domain enclosed by C on S. Prove that

   Area $(D) = \frac{1}{2}\int_C (bz-cy)\,dx + (cx-az)\,dy + (ay-bx)\,dz.$