

NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics
MA 1505 Mathematics I
Tutorial 10

1. A table of values of a function f is given below.

$x \backslash y$	0	1	2
0	1	6	4
1	3	5	7
2	8	2	9

For example, $f(1, 1) = 5$. Find $\int_C \nabla f \bullet d\mathbf{r}$, where

- (a) C has parametric equations $x = t^2 + 1, y = t^3 + t, 0 \leq t \leq 1$.
 (b) C is the unit circle $x^2 + y^2 = 1$.

Ans: (a) 6; (b) 0

2. An 80 kg man carries a pail of water weighing 10 kg up a helical (spiralling) staircase with radius 6 m. The man makes exactly three complete revolutions and reaches a height of 30 m. Suppose a total of 2 kg of water leaks steadily during the man's ascent. Express the work done by the man against gravity $g \text{ ms}^{-2}$ as a line integral $\int_C \mathbf{F} \bullet d\mathbf{r}$ by writing down the vector field \mathbf{F} and the vector equation of the curve C .

Evaluate the work done by computing this line integral.

Ans: $2670g \text{ kg}\cdot\text{m}^2\text{s}^{-2}$

3. Evaluate the line integral $\oint_C xy^2 dx + x^3 dy$, where C is the rectangle with vertices $(0, 0)$, $(2, 0)$, $(2, 3)$ and $(0, 3)$ oriented anticlockwise by (i) direct computation, and (ii) using Green's Theorem.

Ans: 6

4. Evaluate $\oint_C (x^5 - y^5) dx + (x^5 + y^5) dy$, where C is the boundary with positive orientation of the region between the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$, where $0 < a < b$.

Ans: $5\pi(b^6 - a^6)/4$

5. Use Green's Theorem to find the exact value of the line integral

$$\oint_C (xy - \tan(y^2)) dx + (x^2 - 2xy \sec^2(y^2)) dy$$

where C is the positively oriented triangle with vertices at $(0, 0)$, $(1, 0)$, $(0, 2)$.

Ans: $\frac{1}{3}$.

6. Let C denote the upper half of the circle $x^2 + y^2 = 1$ that joins $(-1, 0)$ to $(1, 0)$.

Find the value of the line integral

$$\int_C \{y(x^2 + e^x) + x^2\} dx + (e^x - xy^2) dy.$$

(Suggestion: You may have observed that Green's Theorem is not yet applicable because the curve C is not closed. Try to close it!)