MA1506 TUTORIAL 4

Question 1

Find the equilibrium points for the following equations. Determine whether these equilibrium points are stable, and, if so, find the approximate angular frequency of oscillation around those equilibria.

(i) $\ddot{x} = \cosh(x)$ (ii) $\ddot{x} = \cos(x)$ (iii) $\ddot{x} = \tan(\sin(x))$.

Question 2

Use Graphmatica to graph the solutions of

$$\ddot{x} + x = \cos(\omega t)$$

[with $x(0) = 0$, $\dot{x}(0) = 0$] when $\omega = 1$ and when $\omega = 0.9$. In the second case, verify that the two frequencies apparent in the graph are what you expect them to be. [Please get the solutions from the lecture notes and then graph them — don't use Graphmatica to solve the equations, except to check your answers!

Question 3

Consider a forced damped harmonic oscillator, as in Chapter 2 of the notes. Recall that the amplitude response function is a function of α, the input frequency. Find the maximum of this function. [Note: be careful, there are different answers when the frictional constant b is large and when it is small.] Show that, when b is so small that the dimensionless quantity $b^2/m^2\omega^2$ can be neglected, the maximal amplitude is proportional to $1/b$.

Question 4

A fully loaded large oil tanker can be modelled as a solid object with perfectly vertical sides and a perfectly horizontal bottom, so all horizontal cross-sections have the same area, equal to A. Archimedes' principle [http://en.wikipedia.org/wiki/Buoyancy] states that the upward force exerted on a ship by the sea is equal to the weight of the water pushed aside by the ship. Let ρ be the mass density of seawater, and let M be the mass of the ship, so that its weight is Mg, where g is 9.8 m/sec^2. When the ship is at rest, find the distance d from sea level to the bottom of the ship. This is called the **draught** of the ship.

Suppose now that the ship is *not* at rest; instead it is moving in the vertical direction. Let $d + x(t)$ be the distance from sea level to the bottom of the ship, where d is the draught as above. Show that, if gravity and buoyancy are the only forces acting on the ship, it will bob up and down with an angular frequency given by $\omega = \sqrt{\rho A g/M}$.

Next, suppose that waves from a storm strike the ship [which is initially at rest with $x(0) = 0$] and exert a vertical force $F_0 \cos(\omega t)$ on the ship, where F_0 is the amplitude of the wave force. Let H be the height of the deck of the ship above sea level when the ship is at rest. [We assume that the ship is heavily loaded, so H is much less than d.] Write down a formula which allows you to compute when the ship sinks. [That is, find an equation satisfied by t_{sink}, the time at which the ship's deck first goes under water. You don't need to solve this equation — though of course that can be done by a computer.]