MA1506 TUTORIAL 3

1. Solve the following differential equations:

(a) $y'' + 6y' + 9y = 0, \qquad y(0) = 1, \qquad y'(0) = -1$

(b) $y'' - 2y' + (1 + 4\pi^2)y = 0, \qquad y(0) = -2, \qquad y'(0) = 2(3\pi - 1)$

2. Find particular solutions of the following:

(a) $y'' + 2y' + 10y = 25x^2 + 3$ (b) $y'' - 6y' + 8y = x^2 e^{3x}$

(c) $y'' - y = 2xsin(x)$ (d) $y'' + 4y = sin^2(x)$

3. Use the method of variation of parameters to find particular solutions of

(a) $y'' + 4y = sin^2(x)$ (b) $y'' + y = sec(x)$

4. One of the most important kinds of **nonlinear** second-order odes takes the form $y'' = F(y)$. [That is, y'' can be expressed in terms of y only, with no y'.] These can be solved with the aid of the following clever trick. Use the chain rule to prove that

$$\frac{d^2y}{dx^2} = \frac{d}{dy}[\frac{1}{2}y'^2].$$

Use this trick to show that any equation of the form $y'' = F(y)$ can be reduced to a separable first-order ode — and therefore solved, because we know that we can always solve this kind of equation.

Suppose that the Earth were to stop moving tomorrow. It would of course immediately begin to fall towards the Sun. How long would we have before reaching the orbit of Venus [by which time we would all have been fried]? Information: the radius of the earth's orbit is about 150 billion m, that of Venus' orbit is about 100 billion m [both orbits being approximately circular], the acceleration due to gravity at a distance of r from a central object of mass M is $-GM/r^2$, where M [the mass of the Sun] is about 2×10^{30} kg, and G is Newton's constant $= 6.67 \times 10^{-11}$ in MKS units. So you have to solve the equation $\ddot{r} = -GM/r^2$. Use the above clever trick and solve the resulting first-order separable equation. You will get a nasty definite integral [which **can** actually be done]; go to http://wims.unice.fr/wims/ if you would rather let a computer do it. [The computer may not want to do it either [why?]; you may have to adjust your domain of integration very slightly in order to persuade it to cooperate.]