MA1506 TUTORIAL 1

1. Solve the following differential equations:

(a) $x(x+1)y' = 1$ (b) $(sec(x))y' = cos(5x)$

(c) $y' = e^{(x-3y)}$ (d) $(1+y)y' + (1-2x)y^2 = 0$

Use www.graphmatica.com to sketch the functions you found as solutions of [a]-[d], if y = 1/2 at x = 1. Graphmatica can *also* directly sketch the graph if you insert the equation itself; for example, in [a] you just enter $x(x+1)dy = 1$ $\{1, 1/2\}$. [curly brackets for the initial conditions, with x followed by y.] Here dy represents y'. Use this to check that your answers were correct.

2. Experiments show that the rate of change of the temperature of a small iron ball is proportional to the difference between its temperature $T(t)$ and that of its environment, T_{env} (which is constant). Write down a differential equation describing this situation. Show that $T = T_{env}$ is a solution. Does this make sense? The ball is heated to $300°F$ and then left to cool in a room at $75°F$. Its temperature falls to $200°F$ in half an hour. Show that its temperature will be $81.6°F$ after 3 hours of cooling.

3. In very dry regions, the phenomenon called **Virga** is very important because it can endanger aeroplanes. [See http://en.wikipedia.org/wiki/Virga]. Virga is rain in air that is so dry that the raindrops evaporate before they can reach the ground. Suppose that the volume of a raindrop is proportional to the 3/2 power of its surface area. [Why is this reasonable? Note: raindrops are not spherical, but let's assume that they always have the same shape, no matter what their size may be.] Suppose that the rate of reduction of the volume of a raindrop is proportional to its surface area. [Why is this reasonable?] Find a formula for the amount of time it takes for a virga raindrop to evaporate completely, expressed in terms of the constants you introduced and the initial surface area of a raindrop. Check that the units of your formula are correct. Suppose somebody suggests that the rate of reduction of the volume of a raindrop is proportional to the **square** of the surface area. Argue that this cannot be correct.

4. One theory about the behaviour of moths states that they navigate at night by keeping a fixed angle between their velocity vector and the direction of the Moon [or some bright star; see http://en.wikipedia.org/wiki/Moth]. A certain moth flies near to a candle and mistakes it for the Moon. What will happen to the moth?

Hints: in polar coordinates (r, θ), the formula for the angle ψ between the radius vector and the velocity vector is given by $\tan(\psi) = r\frac{d\theta}{dr}$. [If you want to derive this formula, remember that the tangential component of a small displacement in polar coordinates $(r,\theta) \to (r + dr, \theta + d\theta)$ is r$d\theta$, and the radial component is just dr. Now use simple trigonometry.] Use the formula to solve for r as a function of θ. Note that graphmatica can graph in polar coordinates if you spell out θ [as theta!]

5. Solve the following equations:

(a) $y' = \frac{1-2y-4x}{1+y+2x}$ (b) $y' = \left(\frac{x+y+1}{x+y+3}\right)^2$

(c) $x + y + 1 + (-x + y - 3)y' = 0$