NATIONAL UNIVERSITY OF SINGAPORE

Department of Mathematics

MA4247 Complex Analysis II Homework 4 (due 9 Nov)

1. (a) Show that the cross-ratio of any four distinct points in the complex plane is invariant under any linear fractional transformation T.

   (b) Find all the linear fractional transformations that map the imaginary axis to the unit circle $|w| = 1$ and the point $z = 1$ to ∞.

2. Find an analytic isomorphism from the semi-infinite strip

$$\{z \in \mathbb{C} : \text{Re } z < 0,\ 0 < \text{Im } z < 1\}$$

   to the upper half plane $\{z \in \mathbb{C} : \text{Im } z > 0\}$.

3. (a) Does there exist an *entire* function f such that

$$f(\frac{1}{n}) = \frac{n + n^3}{1 + 3n^4}$$

   for all positive integers n? Justify your answer.

   (b) Show that for any four distinct points $z_1, z_2, z_3, z_4 \in \hat{\mathbb{C}}$, there exists a Möbius transformation T and a complex number k such that $T(z_1) = 1$, $T(z_2) = -1$, $T(z_3) = k$ and $T(z_4) = -k$.