NATIONAL UNIVERSITY OF SINGAPORE

Department of Mathematics

MA4247 Complex Analysis II   Homework 1 (due 24 Aug)

1. Suppose that $f(z)$ has an isolated singularity at z_0, that is, $f(z)$ is analytic on $B(z_0, R) \setminus \{z_0\}$ for some $R > 0$. Suppose further that f is bounded on $B(z_0, R) \setminus \{z_0\}$ (there exists $K > 0$ such that $|f(z)| < K$ for all $z \in B(z_0, R) \setminus \{z_0\}$). Show that z_0 is a removable singularity of f. (Do this directly using the formula for the coefficients b_n's of the Laurent series in Laurent's Theorem, cf Lecture notes, part I, pg. 18).

2. Using question 1, prove the Casorati-Weierstrass Theorem:
Suppose $f(z)$ has an essential singular point at $z = z_0$ and let w_0 be any complex number. Then, for any positive number ε, the inequality

$$|f(z) - w_0| < \varepsilon$$

is satisfied at some point z in each deleted neighborhood $0 < |z - z_0| < \delta$ of z_0 (Hint: Suppose not, use proof by contradiction. It is a good exercise to understand what the negation of the statement in the Theorem means).