NATIONAL UNIVERSITY OF SINGAPORE

Department of Mathematics

MA4247 Complex Analysis II Supplementary Exercise 3

These are some problems, some of which are rather challenging, you may like to try solving them, either individually, or with your group members, and it would be good if you also discuss this on the forum.

1. Prove that $ze^z = a$ where $a \neq 0$ is real has infinitely many roots.
2. Prove that $z \tan z = a$, where $a > 0$ has infinitely many real roots but no imaginary roots.
3. Determine the total number of zeroes of the polynomial

   $$p(z) = i(z^7 - 2z^5) + z^3 - 1$$

   that lie in the upper half plane. Justify your answer.
4. Find the positive integer n such that the polynomial

   $$p(z) = z^5 - 30z^2 + 1$$

   has exactly 3 zeroes (counted with multiplicity) in the annulus

   $$A = \{z \in \mathbb{C} : n < |z| < n + 1\}.$$

   Justify your answer.