

## MA3501 Mid-term Test

### Answer Sheet

Seat Number:

Matriculation Number:

Name (Block Letters) :

Tut Gp (or day & time) :

Write down (in ink) your answers in the boxes provided.

Question	Answer		
	(i)	(ii)	(iii)
1	a=	b=	
2	a=		
3			
Total			

Answer all the questions and enter your answer (in ink only) on the answer sheet.  
Please hand in only your answer sheet.

1. Let  $A = \begin{pmatrix} a & -1 \\ b & 1 \end{pmatrix}$

Given that  $A$  has an eigenvalue 2 of multiplicity 2.

- (i) Find the value of  $a$ . [2 marks]
- (ii) Find the value of  $b$ . [2 marks]
- (iii) Let  $x(t)$  be the solution of the differential equation

$$x'(t) = Ax(t), \quad x(1) = \begin{pmatrix} 2e^2 \\ e^2 \end{pmatrix}$$

find the value of  $x(0)$ . [3 marks]

2. Given that  $A = \begin{pmatrix} 4 & a \\ 3 & 4 \end{pmatrix}$  has an eigenvalue  $4 - 3i$ .

- (i) Find the value of  $a$ . [2 marks]
- (ii) Find the eigenvector corresponding to  $4 - 3i$ . [2 marks]
- (iii) Let  $x(t)$  be the solution of the differential equation

$$x'(t) = Ax(t), \quad x(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

find the value of  $x(\pi)$ . [3 marks]

3. Let  $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$

- (i) Find the eigenvalues of  $A$ . [2 marks]
- (ii) Find the eigenvectors of  $A$ . [2 marks]

(iii) Consider the differential equation  $x'(t) = Ax(t) + \begin{pmatrix} 3 \\ 9 \end{pmatrix}$

find real numbers  $a$  and  $b$  such that  $x(t) = \begin{pmatrix} a \\ b \end{pmatrix}$

is a solution of the differential equation. [2 marks]