NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF MATHEMATICS

SEMESTER 2 EXAMINATION 2012-2013

QF3101 Investment Instruments: Theory and Computation

April/May 2013 — Time allowed : 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains a total of **FIVE (5)** questions and comprises **FIVE (5)** printed pages (including this cover page).
2. Answer **ALL** questions. The mark for each question is indicated at the beginning of the question.
3. Start your answer to each question on a new page.
4. This is a closed book examination. You are allowed to use only **ONE (1)** A4 double-sided handwritten help sheet.
5. You may use **a non-programmable calculator**. However, you should lay out systematically the various steps in the calculations.
6. Final answers to all numerical computations are to be given to the (i) **nearest basis point** for **rates**, (ii) **nearest hundredth of a cent** for **monetary values** and (iii) **four decimal places** for all others.

Question 1 *[20 marks]*

(a) Time now is 0 and a forward contract on a dividend-paying stock expires at time T. The current stock price is S_0 and it has been announced that two dividends of value D_1 and D_2 will be paid at times t_1 and t_2 with $0 < t_1 < t_2 < T$. The theoretical forward price for this contract is given by

$$F_0 = \frac{S_0}{d(0,T)} - \frac{D_1}{d(t_1,T)} - \frac{D_2}{d(t_2,T)}$$

where $d(t_k, T)$ is the discount factor from time T to time t_k.

(i) If the market forward price F_0' is such that

$$F_0' < \frac{S_0}{d(0,T)} - \frac{D_1}{d(t_1,T)} - \frac{D_2}{d(t_2,T)},$$

devise an appropriate arbitrage strategy.

(ii) Suppose for some reasons, a trader who wishes to take up the long position in such a forward contract is not able to find a counter-party. Describe how the trader could replicate the forward contract using cash loans.

(b) Copper futures are trading at CME with expiries at monthly intervals with the last trading day for each contract being the third last business day of the expiry month. Each contract covers 25,000 pounds of copper. The last trading day for Aug'13 futures is 28 Aug 2013, and that for Sep'13 futures is 26 Sep 2013.

Company ABC has signed a contract to deliver 100,000 pounds of copper to a copper fabricator on 30 Aug 2013 at the prevailing spot price on that date.

(i) Suppose today, 29 April 2013, the futures prices for Aug'13 and Sep'13 futures are 3.3735 and 3.3815 USD per pound. Describe the futures position ABC could adopt today to minimize price risk. You can assume that minimum variance hedge is not used by ABC.

(ii) Suppose on 30 Aug 2013, the spot price for copper is 3.4825 USD per pound, and ABC closes out its futures position (taken as in (i)) at price 3.4255 USD per pound. Ignoring margin requirements and the marked-to-market cash flows, what is the effective price ABC is able to achieve? Is the hedge perfect?

Question 2 [*20 marks*]

(a) Company ABC has just issued a 3-year floating rate bond with yearly coupons payable on 1-year LIBOR with payments in arrears. The payments of the coupon are to take place exactly one year, two years and three years from now. ABC wishes to convert this floating-rate liability to a fixed-rate one.

(i) Describe the financial instrument ABC can use to achieve its objective.

(ii) Suppose none of the next three years is a leap year, and the one-year , two-year and three-year LIBORs are currently 2.1%, 2.3% and 2.7% respectively. Determine the fixed rate (quoted on actual/360 basis) ABC can achieve in converting its liability.

(iii) Suppose after the first coupon payment for its floating rate bond, the one-year and two-year LIBORs are 2.8% and 3.2% respectively. What is the value of the financial instrument (your answer to (i)) per $1 notional principal to ABC at this point in time?

(b) Company A wishes to borrow at a fixed rate of interest for 5 years while Company B wishes to borrow at a floating rate of interest for the same 5-year period. They have been quoted the following rates per annum:

	Fixed Rate	**Floating Rate**
Company A	1.4%	LIBOR+1.1%
Company B	1.1%	LIBOR+1.0%

Design a swap arrangement, with a bank acting as intermediary, that will allow the bank to earn 10 basis points and with A and B having the same rate savings. If the current 5-year Treasury yield is 0.10%, how would the bank quote this swap in the swap market?

Question 3 [*25 marks*]

(a) A Treasury bill maturing in 200 days is quoted at a bid of 2.84 and an ask of 2.82. Suppose that you do not spend time waiting in the bid-ask queue and buy this Treasury bill with a $10,000 face value.

(i) How much do you have to pay?

(ii) What is the investment rate associated with this purchase price? Assume 365 days in the year concerned.

(b) Suppose the dollar price of a treasury bill with 145 days to maturity is $98.15, and the price of a treasury bill futures which expires 55 days from now is 99.25 (index basis). Determine the implied repo rate (annualized with continuous compounding).

(c) Currently, LIBORs for various maturities are given below.

m	Maturity (Days)	m-month LIBOR (%)
3	91	1.25
6	182	1.75
9	274	2.15
12	365	2.45
15	456	2.75

(i) Determine the fair fixed rate for a 6×12 forward rate agreement (FRA) configured today.

(ii) A one-year fixed rate bond paying coupons every six months has just been issued with a price equal to its face value. If the LIBOR term structure is applicable for valuation of future cash flows of this bond, what is the coupon rate (annualized on actual/360 basis) of this bond?

Question 4 [*15 marks*]
Consider the two-factor model for asset i, $i = 1, 2$,

$$r_i = a_i + b_{1i} f_1 + b_{2i} f_2 + e_i$$

where $\mathrm{E}(e_i) = 0$, $\mathrm{cov}(e_i, f_1) = \mathrm{cov}(e_i, f_2) = 0$, and $\mathrm{cov}(e_i, e_j) = 0$ for $i \neq j$. The following estimates of the variances and covariances have been obtained:

	Covariance with			
	Asset 1	Asset 2	Factor 1	Factor 2
Factor 1	0.35	0.02	0.81	0.51
Factor 2	−0.25	0.15	0.51	0.64

(i) Determine the factor loadings for both assets.

(ii) If the risk free rate is 1%, and the expected rates of return for assets 1 and 2 are 2% and 4% respectively, determine the factor prices of both factors.

Question 5 *[20 marks]*

(a) In a historical simulation using 250 past daily returns, $k = 1, 2, \ldots, 250$, the corresponding hypothetical daily portfolio changes were obtained and sorted in ascending order. The 8 most negative portfolio changes and their age (the k value) are shown in the following table.

Age k	Change in Portfolio Value
7	−273560
28	−245910
51	−234850
62	−229320
31	−226555
53	−223790
42	−222131
12	−221025

For an age-weighted historical simulation with the weight for age k being

$$w_k = 0.03\lambda^{k-1}, \qquad \lambda = 0.97,$$

determine the 1-day 95% absolute VaR.

(b) A treasury bill with six months to expiry has a face value of $10,000.

(i) Given the following information, use cash flow mapping to determine an "equivalent" portfolio of zeros with maturities of three months and one year.

Standard Maturity	Yield (%)	Price Volatility (daily %)	Correlation Matrix	
			3 months	1 year
3 months	0.58	0.12	1	0.98
1 year	0.88	0.16	0.98	1

(ii) Determine the 5-day 99% VaR of this treasury bill position.

END OF PAPER