NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF MATHEMATICS

SEMESTER 2 EXAMINATION (2012–2013)

MA1521 Calculus for Computing

May 2013 — Time allowed : 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains a total of **EIGHT (8)** questions and comprises **FOUR (4)** printed pages.

2. Answer **ALL** questions. The marks for questions are not necessarily the same; marks for each question are indicated at the beginning of the question.

3. Candidates may use calculators. However, they should lay out systematically the various steps in the calculations.

Question 1 [10 marks]

Let $f(x, y) = e^x(2xy + y^2)$.

(i) Find the coordinates of all the critical points of f.

(ii) Determine whether the critical points of f are local maximums, local minimums or saddle points. Justify your answers.

Question 2 [10 marks]

Let $P(x_0, y_0, z_0)$ be a point different from the origin. Using the Lagrange multipliers method, find the point on the unit sphere $x^2 + y^2 + z^2 = 1$ which is closest to P.

Question 3 [16 marks]

Find the following indefinite integrals.

(a) $\displaystyle\int \frac{(\ln x)^3}{x^2}\,dx.$

(b) $\displaystyle\int \frac{12}{x^4 - 5x^2 + 4}\,dx.$

Question 4 [12 marks]

Determine whether each of the series is convergent or divergent. Justify your answers.

(a) $\displaystyle\sum_{n=0}^{\infty}(-1)^{n+1}\frac{n+1}{n^2+1}.$

(b) $\displaystyle\sum_{n=1}^{\infty}\left(\sqrt[3]{n^2+n} - \sqrt[3]{n^2-n}\right).$

Question 5 [10 marks]

For each $n \geq 0$, define

$$I_n = \int_{-\infty}^{\infty} x^n e^{-x^2}\, dx.$$

(i) Show that for all $n \geq 2$, $2I_n = (n-1)I_{n-2}$.

(ii) Using the result of (i) and the fact that

$$\int_{-\infty}^{\infty} e^{-x^2}\, dx = \sqrt{\pi},$$

evaluate

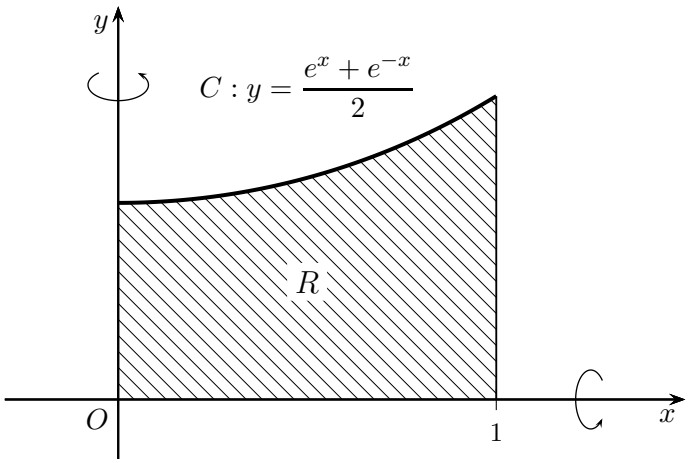
$$\int_{-\infty}^{\infty} x^4 e^{-x^2/2}\, dx.$$

Question 6 [20 marks]

Let C be the curve defined by

$$y = \frac{e^x + e^{-x}}{2}, \qquad 0 \leq x \leq 1,$$

and let R denote the region enclosed by the curve C, $x = 1$ and the axes.



(i) Find the volume of the solid formed by rotating the region R about the x-axis.

(ii) Find the volume of the solid formed by rotating the region R about the y-axis.

(iii) Find the length of the curve C.

(iv) Find the area of the surface formed by rotating the curve C about the x-axis.

Question 7 [16 marks]

Solve the following differential equations.

(a) $x\dfrac{dy}{dx} + y = xy^2 \ln x, \quad y = 1$ when $x = 1$.

(b) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = \dfrac{e^{-x}}{x} \quad (x > 0), \quad y = 0$ when $x = 1$ and $x = e$.

Question 8 [6 marks]

Show that the series

$$\sum_{n=1}^{\infty} \frac{x^{n-1}}{(1-x^n)(1-x^{n+1})} = \begin{cases} \dfrac{1}{(1-x)^2}, & \text{if } |x| < 1, \\ \dfrac{1}{x(1-x)^2}, & \text{if } |x| > 1. \end{cases}$$

End of Paper