NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF MATHEMATICS
SEMESTER 1 EXAMINATION 2012-2013

MA2101 Linear Algebra II

November 2012 – Time allowed : 2 hours

INSTRUCTIONS TO CANDIDATES

(1) Answer all questions.
(2) There are SIX (6) questions in this paper. Each is worth TEN (10) points.
(3) This examination paper consists of THIRTEEN (13) printed pages.
(4) **Write all answers on this question paper**.
(5) Provide proofs/justification for all results used other than those from the lectures or tutorials.

Matric No: ______________________________

Question	1	2	3	4	5	6
Points						

Total	

1. (a) Given that $V = \{p(x) \in P_2(\mathbb{R}) : p(1) = 0\}$ is a subspace of $P_2(\mathbb{R})$, find a basis for V.

(b) If V is endowed with the inner product

$$\langle p(x), q(x) \rangle = \int_0^1 p(x)q(x)\,dx,$$

find an orthonormal basis for V.

2. Let $A = \begin{pmatrix} 2 & 0 \\ -3 & 1 \end{pmatrix}$ and let $T : M_2(\mathbb{R}) \to M_2(\mathbb{R})$ be the linear transformation defined by

$$T(B) = AB.$$

Determine if T is diagonalizable. If so, find an ordered basis β for $M_2(\mathbb{R})$ so that $[T]_\beta$ is a diagonal matrix.

More working space for Q2

3. Let V be a finite dimensional complex inner product space. Suppose that there are orthonormal bases β and γ for V so that $[T]_\beta^\gamma$ is a unitary matrix. Show that T is a unitary linear transformation.

More working space for Q3

4. Let V and W be finite dimensional vector spaces and let T and S be one-to-one linear transformations from V to W so that $R(T) \cap R(S) = \{0\}$. If $\{v_1, \ldots, v_n\}$ is a basis for V, show that $\{(T + S)(v_1), \ldots, (T + S)(v_n)\}$ is a basis for $R(T + S)$.

More working space for Q4

5. Let $T : V \to V$ be a linear transformation on a finite dimensional inner product space.
(a) Show that λ is an eigenvalue of T if and only if $\overline{\lambda}$ is an eigenvalue of T^*.

(b) Show that $E_\lambda(T) = [R(T^* - \overline{\lambda} I)]^\perp$ and deduce that $\dim(E_\lambda(T)) = \dim(E_{\overline{\lambda}}(T^*))$.

6. Let V be an n-dimensional real vector space. Suppose that $f_1, \ldots, f_m$ are linearly independent elements of $L(V, \mathbb{R})$. Show that

$$\dim(\bigcap_{i=1}^{m} N(f_i)) = n - m.$$

Hint: Consider the linear transformation $T : V \to \mathbb{R}^m$ given by $T(v) = (f_1(v), \ldots, f_m(v))$.

More working space for Q6

END