NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF MATHEMATICS

SEMESTER 2 EXAMINATION 2011-2012

QF3101   Investment Instruments: Theory and Computation

April/May 2012 — Time allowed : 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains a total of **FIVE (5)** questions and comprises **FIVE (5)** printed pages (including this cover page).

2. Answer **ALL** questions. The mark for each question is indicated at the beginning of the question.

3. Start your answer to each question on a new page.

4. This is a closed book examination. You are allowed to use only **ONE (1)** A4 double-sided handwritten help sheet.

5. You may use a non-programmable calculator. However, you should lay out systematically the various steps in the calculations.

6. Express all numerical answers up to **4 decimal places**.

Question 1   [*25 marks*]

(a) A trader wishes to lock in the purchase price of a portfolio of three Treasury bills, namely TB1, TB2 and TB3 when they are newly issued by the Treasury 3 months later. TB1 is a 3-month Treasury bill of $10,000 face value, TB2 is a 6-month Treasury bill of $20,000 face value and TB3 is a 1-year Treasury bill of $50,000 face value. The trader decides that a forward contract is to be used to achieve this objective. The current 3-month, 6-month, 9-month, 12-month, 15-month spot rates are 0.8%, 1.2%, 1.5%, 1.8% and 2.2% (annualized, quarterly compounding) respectively.

   (i) Determine the key terms of the forward contract which the trader could use to achieve its objective.

   (ii) Determine the forward price for this contract.

   (iii) Suppose three months later, immediately before the delivery of the Treasury bills, the 3-month, 6-month, 9-month, 12-month, 15-month spot rates are 1.2%, 1.6%, 2.0%, 2.4% and 3.0% (annualized, quarterly compounding) respectively. What is the value of the trader's forward position?

(b) Time now is 0 and a one-year forward contract is initiated on 5 tonnes of coffee bean. The storage cost for 5 tonnes of coffee bean is C_1 for the first six months, and C_2 for the second six months, payable at the start of each six-month period. Suppose the spot price of coffee bean is S_0 ($/tonne), and $d(t_1, t_2)$ is the discount factor between two time points, $t_1 < t_2$, $t_1 \in [0, 1 \text{ year}]$ and $t_2 \in [0, 1 \text{ year}]$.

   (i) Obtain the forward price F_0 in terms of S_0, C_1, C_2 and the appropriate discount factors.

   (ii) If the market quotation for the forward price F_0 is less than that given by the expression in (b)(i), give an arbitrage strategy that an arbitrageur could adopt.

Question 2   [*25 marks*]

(a) A 1-year interest rate swap is to be configured on quarterly in-arrear exchanges of a fixed rate payment for a floating level payment linked to a 3-month floating rate on a notional principal of $50,000. The current 3-month, 6-month, 9-month and 12-month spot rates are 1.2%, 1.4%, 1.6% and 1.8% (annualized, quarterly compounding) respectively.

   (i) Determine the swap rate of the 1-year interest rate swap.

   (ii) The 1-year interest rate swap can be viewed as a portfolio of four forward contracts. Determine the values of the first and the last to expire forward contracts at the inception of the swap for the fixed rate payer.

(to be continued)

(iii) Suppose six months later, immediately after the 2nd swap exchange for the interest rate swap, the 3-month, 6-month, 9-month and 12-month spot rates are 1.4%, 1.6%, 1.8% and 2.0% (annualized, quarterly compounding) respectively. Determine the swap value of this interest rate swap for the fixed rate payer.

(b) Company A wishes to borrow at a fixed rate of interest for 3 years while Company B wishes to borrow at a floating rate of interest for the same 3-year period. They have been quoted the following rates per annum:

	Fixed Rate	Floating Rate
Company A	2.48%	LIBOR+1.15%
Company B	2.21%	LIBOR+1.0%

Design a swap arrangement, with a bank acting as intermediary, that will allow the parties involved to share the rate savings equally.

Question 3 *[20 marks]*

(a) Suppose the dollar price of a treasury bill (TB1) with 60 days to maturity is $99.25, and that of another treasury bill (TB2) with 150 days to maturity is $98.31.

  (i) Determine the discount yield and investment rate for TB1. Assume 365 days in the year concerned.

  (ii) Determine the futures price (index basis) of a treasury bill futures which expires 60 days from now and delivers a 90-day treasury bill.

  (iii) If the actual futures price (index basis) is higher than the theoretical futures price obtained in (a)(ii), briefly describe the arbitrage strategy that an arbitrageur could adopt.

(b) Suppose today is 18 June 2012, and the following table gives a summary of information on six Eurodollar futures with prices recorded at the end of the day.

Contract	Daily closing Price	Contract Expiry Date	#Days from 18-June to Expiry Date
Jun 12	99.530	18-Jun-2012	0
Sep 12	99.510	17-Sep-2012	91
Dec 12	99.480	17-Dec-2012	182
Mar 13	99.450	18-Mar-2013	274
Jun 13	99.395	17-Jun-2013	365
Sep 13	98.320	16-Sep-2013	456

(to be continued) ...–4–

(i) Determine the fair fixed rate for a 3×9 forward rate agreement initiated today.

(ii) A bank has a loan contract to lend a client a sum of money for a 9-month period starting 6 months from now. What rate (annualized, actual/360) should the bank quote the client if it wants to be assured of an earning of 100 basis points?

Question 4 [*10 marks*]

Consider the three-factor model for asset i, $i = 1, 2$,

$$r_i = a_i + b_{1i}f_1 + b_{2i}f_2 + b_{3i}f_3 + e_i$$

where $\mathrm{E}(e_i) = 0$, $\mathrm{cov}(e_i, f_1) = \mathrm{cov}(e_i, f_2) = \mathrm{cov}(e_i, f_3) = 0$, and $\mathrm{cov}(e_i, e_j) = 0$ for $i \neq j$.

(i) Derive an expression for the variance of r_i in terms of factor loadings and the variances and covariances of the three factors. Identify the systematic risk component involved.

(ii) The following estimates of the variances and covariances have been obtained:

	Covariance with				
	Asset 1	Asset 2	Factor 1	Factor 2	Factor 3
Factor 1	0.25	0.05	0.62	0	0.32
Factor 2	0.15	0.12	0	0.12	0
Factor 3	0.08	0.21	0.32	0	0.88

Determine the factor loadings for asset 2.

Question 5 [*20 marks*]

(a) Consider a portfolio consisting of four options, namely OP1, OP2, OP3 and OP4. Options OP1 and OP2 have 1 share of stock A as the underlying asset while OP3 and OP4 have 1 share of stock B as the underlying asset. The delta's of OP1, OP2, OP3 and OP4 are 1.2, 2.3, -1.5, 0.5 respectively, and the current unit prices for stocks A and B are \$12.55 and \$6.87 respectively. Assume that the assets' returns are normally distributed, the daily volatilities of the stocks A and B are 4.8%, 2.1% respectively, and that the coefficient of correlation between their returns is $\rho = 0.75$. For a risk horizon of 10 days, a year of 250 trading days and a confidence level of 95%, determine the diversified VaR of the portfolio.

(b) A treasury bond with five months to expiry has a face value of $50,000.

    (i) Given the following information, use cash flow mapping to map this bond to a portfolio of zeros with maturities of three months and one year.

Standard Maturity	Yield (%)	Price Volatility (daily %)	Correlation Matrix	
			3 months	1 year
3 months	1.28	0.10	1	0.96
1 year	1.08	0.14	0.96	1

    (ii) Determine the 10-day 95% VaR of this treasury bond position.

END OF PAPER