NATIONAL UNIVERSITY OF SINGAPORE

FACULTY OF SCIENCE

SEMESTER 2 EXAMINATION 2011-2012

MA2214   COMBINATORIAL ANALYSIS

May 2012 — Time allowed : 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains a total of **TEN (10)** questions and comprises **THREE (3)** printed pages.

2. The marks for each question are indicated at the beginning of the question.

3. Candidates may use calculators. However, they should lay out systematically the various steps in the calculations.

Question 1.   [15 marks]

How many different arrangements are there of the letters A, B, C, D, E, F in which

(a) A and B are next to each other and C and D are also next to each other?

(b) E is not the last letter?

(c) A is before B?

(d) A is before B and B is before C?

(e) A is before B and C is before D?

Indicate clearly which answer corresponds to which part and leave your final answers as integers.

Question 2.   [15 marks]
Find the number of ways of distributing 20 apples, 30 bananas and 40 oranges to 8 children if

(a) there is no restriction;

(b) every child has at least 1 apple, 1 banana and 1 orange;

(c) every child has at least 1 fruit;

(d) only 2 particular children are not given any fruit.

[Note that fruits of the same kind are considered as identical objects.]

Question 3.   [5 marks]   Find $\displaystyle\sum_{k=1}^{n} \frac{k}{n^k}\binom{n}{k}$.

Question 4.   [10 marks]
Find the number of positive integers between 1 and 2300 inclusive that are relatively prime to 700.

Question 5.   [5 marks]
In how many ways can 20 objects be selected from five types of objects if the first type can only be selected in multiples of 5, the second can only be selected in multiples of 3, at most 4 of the third type can be selected, at least 3 of the fourth type must be selected, and at most 2 of the fifth type can be selected?

Question 6. [10 marks] Twelve female workers and ten male workers are to be assigned to work in one of four different departments of a company. In how many ways can this be done if

  (a) there is no restriction?

  (b) each department gets at least one worker?

  (c) each department gets at least one female worker?

  (d) each department gets at least one female worker and at least one male worker?

Question 7. [10 marks]
In how many ways can one arrange the letters in the word CORRESPONDENTS so that there is no pair of consecutive identical letters?

Question 8. [10 marks]
A computer program generated 175 distinct positive integers at random, none of which had a prime divisor larger than 10. Prove that we can always find three numbers among them whose product is the cube of an integer.

Question 9. [10 marks]
Use Ferrers diagrams to show that the number of partitions of n is equal to the number of partitions of $2n$ into n parts.

Question 10. [10 marks]
For each positive integer n, let a_n denote the number of n-digit integers formed by the five numbers $1, 2, 3, 4$ and 5 which do not contain consecutive 3's.

  (a) Find a recurrence relation for a_n with the necessary initial conditions.

  (b) Find a_n in terms of n for each n.

<center>END OF PAPER</center>