NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF MATHEMATICS

SEMESTER 2 EXAMINATION (2011–2012)

MA1521 Calculus for Computing

April/May 2012 — Time allowed : 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains a total of **EIGHT (8)** questions and comprises **FOUR (4)** printed pages.
2. Answer **ALL** questions. The marks for questions are not necessarily the same; marks for each question are indicated at the beginning of the question.
3. Candidates may use calculators. However, they should lay out systematically the various steps in the calculations.

Question 1 [10 marks]

Let $f(x, y) = xy(3 - x - y)$.

(i) Find the coordinates of all the critical points of f.

(ii) Classify the local maximums, local minimums and saddle points of f.

Question 2 [10 marks]

An open rectangular box having a volume of 108 cm^3 is to be constructed from cardboard. Find the dimensions of such a box if the amount of cardboard used in its construction is to be minimized. (You may assume that the minimum exists without proof.)

Question 3 [14 marks]

Find the following integrals.

(a) $\displaystyle\int \frac{1}{(x^2+1)(x+1)^2}\,dx.$

(b) $\displaystyle\int \frac{1}{\sqrt{x^2+4x+7}}\,dx.$

Question 4 [12 marks]

Determine whether each of the series is convergent or divergent. Justify your answers.

(a) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\left(\sqrt{n^2+n}-\sqrt{n^2-n}\right).$

(b) $\displaystyle\sum_{n=1}^{\infty}\frac{\ln n}{n\sqrt{n}}.$

Question 5 [12 marks]

(a) Find the radius of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(3n)!}{(n!)^3} x^n$.

(b) Let $f(x) = x^3 \sin(2x^2)$.

(i) Find the Maclaurin series of $f(x)$.

(ii) Hence or otherwise, evaluate $f^{(1521)}(0)$.

Question 6 [16 marks]

Solve the following differential equations.

(a) $x\,\dfrac{dy}{dx} + (x-2)y = 3x^3 e^{-x} \quad (x > 0), \quad y = 0$ at $x = 1$.

(b) $\dfrac{d^2y}{dx^2} - 3\,\dfrac{dy}{dx} + 2y = \dfrac{1}{1+e^{-x}}$.

Question 7 [8 marks]

Suppose that the equation $F(x, y, z) = 0$ implicitly defines each of the three variables x, y and z as functions of the other two:

$$z = f(x, y), \quad y = g(z, x), \quad x = h(y, z).$$

If F is differentiable and $\dfrac{\partial F}{\partial x}$, $\dfrac{\partial F}{\partial y}$ and $\dfrac{\partial F}{\partial z}$ are all nonzero, show that

$$\left(\frac{\partial x}{\partial y}\right)\left(\frac{\partial y}{\partial z}\right)\left(\frac{\partial z}{\partial x}\right) = -1.$$

Question 8 [18 marks]

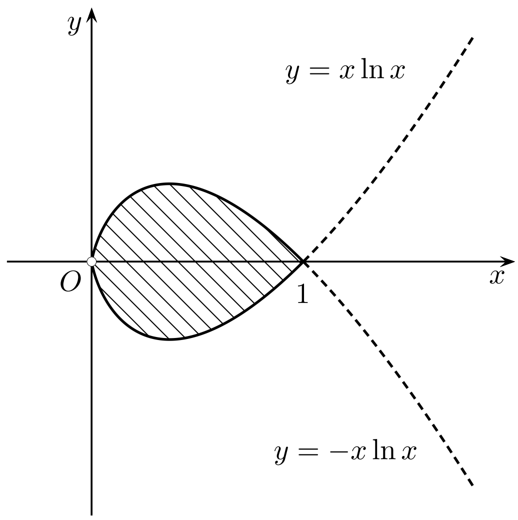
Consider curves $y = x \ln x$ and $y = -x \ln x$, $0 < x \leq 1$, and the region between them.

(a) Write the arc length of the loop as an integral. (You do not need to evaluate it.)

(b) Write the surface area formed by revolving the loop about the x-axis as an integral. (You do not need to evaluate it.)

(c) Find the volume of the solid formed by revolving the region about the y-axis.

(d) Find the volume of the solid formed by revolving the region about the x-axis.



End of Paper