NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF MATHEMATICS

SEMESTER 1 EXAMINATION (2011–2012)

MA1521 Calculus for Computing

November 2011 — Time allowed : 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains a total of **EIGHT (8)** questions and comprises **FOUR (4)** printed pages.
2. Answer **ALL** questions. The marks for questions are not necessarily the same; marks for each question are indicated at the beginning of the question.
3. Candidates may use calculators. However, they should lay out systematically the various steps in the calculations.

Question 1 [10 marks]

Let $f(x, y) = (1 - x)(4y - y^2)$.

(i) Find the coordinates of all the critical points of f.

(ii) Classify the local maximums, local minimums and saddle points of f.

Question 2 [10 marks]

Using the method of **Lagrange multiplier** or otherwise, show that the shortest distance from the point $P(x_0, y_0, z_0)$ to a point (x, y, z) on the plane $ax + by + cz = d$ is given by

$$\frac{|ax_0 + by_0 + cz_0 - d|}{\sqrt{a^2 + b^2 + c^2}}.$$

You may assume that the shortest distance exists and $a^2 + b^2 + c^2 \neq 0$.

Question 3 [14 marks]

Find the following integrals.

(a) $\displaystyle\int (\sin^{-1} x)^2 \, dx$.

(b) $\displaystyle\int \frac{-5x + 2}{x(x - 1)(x^2 + 2)} \, dx$.

Question 4 [16 marks]

Solve the following differential equations.

(a) $\displaystyle\frac{dy}{dx} + y \cot x = 5e^{\cos x} \quad (0 < x < \pi), \quad y = -4 \text{ at } x = \frac{\pi}{2}$.

(b) $\displaystyle\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = e^x, \quad y = 2 \text{ and } \frac{dy}{dx} = 4 \text{ at } x = 0$.

Question 5 [12 marks]

Determine whether each of the following series is convergent or divergent.

(a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \ln\left(\frac{n+1}{n}\right)$.

(b) $\displaystyle\sum_{n=2}^{\infty}\left(\frac{n+1}{n\sqrt{n}+1}-\frac{n-1}{n\sqrt{n}-1}\right)$.

Question 6 [10 marks]

Consider the function $f(x)=\dfrac{x}{x^2+7x+6}$.

(i) Find the Maclaurin series of $f(x)$ and determine its radius of convergence.

(ii) Use the result of (i) to show that $f^{(2011)}(0)=\dfrac{2011!}{5}\left(1-\dfrac{1}{6^{2011}}\right)$.

Question 7 [12 marks]

(i) Let $z=f(x,y)$ be a function such that its second partial derivatives are continuous and $x=x(s,t)$ and $y=y(s,t)$ are functions such that their partial derivatives exist.

Use the Chain Rule for multi-variable function to show that

$$\frac{\partial^2 z}{\partial t^2}=\frac{\partial^2 z}{\partial x^2}\left(\frac{\partial x}{\partial t}\right)^2+2\,\frac{\partial^2 z}{\partial x\,\partial y}\,\frac{\partial x}{\partial t}\,\frac{\partial y}{\partial t}+\frac{\partial^2 z}{\partial y^2}\left(\frac{\partial y}{\partial t}\right)^2.$$

(ii) Let $z=f(x,y)$ be a function such that its second derivatives are continuous and $x=e^s\cos t$ and $y=e^s\sin t$ are functions in s and t.

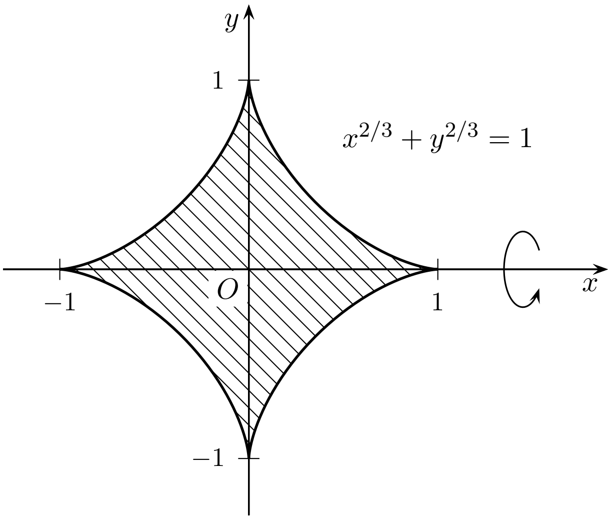
Suppose $\dfrac{\partial^2 z}{\partial x^2}+\dfrac{\partial^2 z}{\partial y^2}=0$. Using the result in (i) or otherwise, show that

$$\frac{\partial^2 z}{\partial s^2}+\frac{\partial^2 z}{\partial t^2}=0.$$

Question 8 [16 marks]

(a) Find the arc length of the curve defined by $x^{2/3} + y^{2/3} = 1$.

(b) Let R be the region enclosed by the curve $x^{2/3} + y^{2/3} = 1$. Find the volume of the solid generated by rotating R about the x-axis.



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