NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF MATHEMATICS

SEMESTER EXAMINATION FOR THE DEGREE OF B.SC.

SEMESTER 2 EXAMINATION 2010–2011

MA2213 Numerical Analysis I

April 2011– Time allowed : 2 hours

Instructions to Candidates

1. This examination paper contains a total of **Five (5)** questions and comprises **Three (3)** printed pages.
2. Answer **ALL** questions.
3. Candidates may use calculators. However, they should lay out systematically the various steps in the calculations.
4. All questions carry equal marks.

Question 1 [20 marks]

Solve the linear system

$$\begin{aligned} 0.003000x_1 + 59.14x_2 &= 59.17 \\ 5.291x_1 - 6.130x_2 &= 46.78, \end{aligned}$$

using a four-digit arithmetic with rounding by

(a) Gaussian elimination, (b) Gaussian elimination with partial pivoting.

Question 2 [20 marks]

(a) Assume that the polynomial with minimal degree interpolating a function $f(x)$ at the points $x_0 = -2$, $x_1 = 2$, $x_2 = 6$, $x_3 = 8$, $x_4 = -12$ and $x_5 = 21$ is

$$P(x) = 111x^4 + 5x^2 - 45x + 15.$$

Find the polynomial with minimal degree interpolating the function $f(x) - 165x^5 + 0.0025x^4 - 986x^3 - 321x^2 + 0.0001$ at x_0, x_1, x_2, x_3, x_4 and x_5.

(b) Let

$$f(x) = -2x^7 + 54321x^5 + 50x^3 + 2000x^2 - 10x + 0.000001,$$

and

$$\begin{aligned} &x_0 = 0.1,\ x_1 = 0.2,\ x_2 = 0.25,\ x_3 = 0.4,\ x_4 = 0.46,\ x_5 = 0.48, \\ &x_6 = 0.50,\ x_7 = 0.52,\ x_8 = 0.6,\ x_9 = 0.7,\ x_{10} = 0.8,\ x_{11} = 1.25. \end{aligned}$$

Find a polynomial $P(x)$ of degree at most 11 that interpolates $f(x)$ at x_i $(i = 0, 1, \cdots, 11)$.

Question 3 [20 marks]

Assume that the formula

$$I(f) = \sum_{i=0}^{6} A_i f(x_i)$$

approximating $\int_{-1}^{1} f(x)dx$ is exact for all polynomials of degree at most 6 and the distinct nodes x_i $(i = 0, 1, \cdots, 6)$ are symmetrically placed about the origin. Let $p(x)$ be a polynomial of degree at most 7, compute the error

$$E(p(x)) = \int_{-1}^{1} p(x)dx - \sum_{i=0}^{6} A_i p(x_i).$$

Question 4 [20 marks]

(a) Let $f(x)$ be a given function. Assume that the polynomial $P(x)$ interpolating the function $f(x)$ at the points $x_0 = -2$, $x_1 = -1$, $x_2 = 0$, $x_3 = 1$, $x_4 = 2$, $x_5 = 4$, $x_6 = 6$ and $x_7 = 8$ is of the form

$$P(x) = -10 + 3(x - x_0) - 5(x - x_0)(x - x_1) + 8(x - x_0)(x - x_1)(x - x_2) - 15\Pi_{i=0}^{6}(x - x_i).$$

Compute

$$f[x_6, x_3, x_5, x_2, x_1, x_4, x_0, x_7], \quad f[x_6, x_3, x_5, x_2, x_1, x_4, x_0],$$

and

$$f[x_6, x_3, x_5, x_2, x_1, x_4, x_7], \quad f[x_6, x_3, x_5, x_2, x_1, x_4].$$

(b) The polynomial $q(x) = 2x^4 + x^3 - x^2 + 5$ assumes the following values:

x	0	1	2	3	4	5
$q(x)$	f_0	f_1	f_2	f_3	f_4	f_5

Find a polynomial $p(x)$ with degree at most 5 without computing f_i $(i = 0, 1, 2, 3, 4, 5)$ that takes the following values:

x	1	2	3	4	5	6
$p(x)$	f_1	f_2	f_3	f_4	f_5	10

Question 5 [20 marks]

(a) How large must n be if the composite trapezoidal rule is used to approximate $\int_0^2 e^x dx$ with a relative error at most 10^{-4}?

(b) How large must n be if the composite Simpson's rule is used to approximate $\int_0^2 e^x dx$ with a relative error at most 10^{-4}?

— END OF PAPER —