NATIONAL UNIVERSITY OF SINGAPORE

FACULTY OF SCIENCE

SEMESTER 1 EXAMINATION 2010-2011

MA1521 Calculus for Computing

November/December 2010 — Time allowed : 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains a total of **EIGHT (8)** questions and comprises **FIVE (5)** printed pages.
2. Answer **ALL** 8 questions. The marks for each question are indicated at the beginning of the question. The maximum score is **80 marks**.
3. **Write your matriculation number neatly on the front page of the answer booklet provided.**
4. **Write your solutions in the answer booklet. Begin your solution to each question on a new page.**
5. Calculators may be used. However, you should lay out systematically the various steps in your calculations.
6. **This is a CLOSED BOOK examination. One A4-sized helpsheet is allowed.**

Question 1 [10 marks]

(a) Let L be the tangent to the curve

$$3x^2 \ + \ 2y^3 \ = \ 5x$$

at the point $(1, 1)$. Find the x-intercept of L.

(b) Let

$$f(x) \ = \ \begin{cases} \dfrac{e^{6x} - 1 - 6x}{x^2} & \text{if } x \neq 0 \\ \\ 18 & \text{if } x = 0 \end{cases}$$

Determine whether f is continuous at $x = 0$. Justify your answer.

Question 2 [10 marks]

(a) Using a substitution, or otherwise, find the value of the integral

$$\int_0^2 \frac{1}{e^x + 1}\, dx.$$

(*Give the exact value in terms of* e.)

(b) The region R lies in the first quadrant, below the line $y = 1$, and is bounded by the graphs of $y = x^2$ and $y = x^4$. Find the volume of the solid generated when R is revolved about the line $y = 1$.

(*Give the exact volume in terms of* π.)

Question 3 [10 marks]

(a) If the kth partial sum s_k of an infinite series $\sum_{n=1}^{\infty} a_n$ is given by

$$s_k = \frac{3^{k+1} - 2k - 3}{3^k}, \quad k \geq 1,$$

find a_n, giving the answer in terms of n.

(b) Find all possible values of the positive constant k such that the series

$$\sum_{n=1}^{\infty} \ln\left(1 + \frac{1}{n^k}\right)$$

is convergent. Justify your answer.

Question 4 [10 marks]

(a) Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{1}{3^n}(2x-1)^n.$$

(b) Using the Maclaurin series of $x\ln(1+x)$, find the exact value of

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2^{2n}\, n(n+2)}\,.$$

Question 5 [10 marks]

(a) The line L is the intersection of the two planes Π_1 and Π_2, where

$$\Pi_1 : \quad x - y + 3z = 1, \qquad \Pi_2 : \quad 2x - y + 2z = 10.$$

Find the point of intersection of L with the xy-plane.

(b) Let $T(x, y)$ be a differentiable function such that $T(0, 0) = 1521$,

$$\frac{\partial T}{\partial x} = x - y \quad \text{and} \quad \frac{\partial T}{\partial y} = y - x.$$

Suppose $T(x, y)$ denotes the temperature at the point $P(x, y)$, whose coordinates are given by

$$x = t^2, \quad y = t^3, \quad \text{where } t \text{ is a real number.}$$

Find the values of t which give local maximum and local minimum temperatures.

Question 6 [10 marks]

(a) Find the local maximum, local minimum and saddle points (if any) of

$$f(x, y) = x^3 + 3xy - y^3 + 8.$$

(b) A rectangular box *without* a top cover is to be made from 48 m^2 of metal. Use Lagrange multipliers to find the largest volume of such a box.

Question 7 [10 marks]

Solve the following differential equations:

(a) $\dfrac{dy}{dx} = 8x^7y$ for $y > 0$, given that $y = e^2$ when $x = 1$.

(b) $\dfrac{dy}{dx} = \dfrac{x^3 + 4x^2y + 2y^3}{x^3 + xy^2}$, where $x > 0$, $y > 0$.

(*Hint: Try a substitution* $y = vx$.)

Question 8 [10 marks]

Solve the following differential equations:

(a) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} - 3y = 8e^x$.

(b) $\dfrac{d^2y}{dx^2} + \left(\dfrac{1-2x}{x}\right)\dfrac{dy}{dx} = 16e^{2x}$, where $x > 0$,

given that $y = 2e^2$ and $\dfrac{dy}{dx} = 8e^2$ when $x = 1$.

END OF PAPER