NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF MATHEMATICS

SEMESTER 2   EXAMINATION   2009–2010

MA3501   Mathematical Methods In Engineering

April 2010– Time allowed : 2 hours

Instructions to Candidates

1. This examination paper contains a total of **Five (5)** questions and comprises **Three (3)** printed pages.

2. Answer **ALL** questions. The marks for the questions are not necessarily the same; the mark for each question is indicated in the question.

3. Candidates may use calculators. However, they should lay out systematically the various steps in the calculations.

Question 1 [20 marks]

Solve the following first order PDE using the method of characteristics for all $(x, y) \in \mathbf{R}^2$:

$$y^2 u_x + x^2 u_y = y^5,$$
$$u(x, 1) = e^{-x^2}.$$

Question 2 [20 marks]

Solve the following initial-value problem for $t > 0$

$$\dot{x} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix} x + \begin{pmatrix} e^t \\ e^{2t} \\ te^{3t} \end{pmatrix},$$

$$x(0) = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}.$$

Question 3 [20 marks]

(i) Let the Fourier transform $F[f]$ of the function $f : \mathbf{R} \to \mathbf{R}$ be given by

$$F[f](\xi) = \begin{cases} 8, & \text{if } \xi = 0, \\ \frac{2\sin(4\xi)}{\xi} & \text{if } \xi \neq 0. \end{cases}$$

Compute

$$F[f(x - 2)], \quad F[e^{-3ix} f(x - 3)].$$

(ii) Let

$$f(x) = \begin{cases} e^{-x}, & \text{if } x \geq 0, \\ 0, & \text{if } x < 0. \end{cases}$$

Compute the Fourier transform $F[f]$.

(iii) Compute the Fourier inverse transform of $\frac{1}{1+i\xi} \frac{\sin(2\xi c)}{\xi}$, where $c \in \mathbf{R}$ is a constant.

Question 4 [25 marks]

(i) Let

$$f(z) = \bar{z} + \frac{1}{z} + z + z^2, \quad \forall z \in \mathbf{C}.$$

Show that $f(z)$ is not analytic at any $z \in \mathbf{C}$.

(ii) Let C denote the circle $|z| = 3.208$ in the counterclockwise direction. Suppose that $f : \mathbf{C} \to C$ is a function which is analytic everywhere in $\{z \in \mathbf{C} | \ |z| \leq 3.208\}$, and

$$f(2i) = 4, \ f'(2i) = 3i, \ f''(2i) = 1.$$

Compute

$$\int_{-C} \left[\frac{1}{z - 2 - i} - \frac{\cos(\pi z)}{z^2 + 3z - 4} + \frac{zf(z)}{2(z - 2i)^3} \right] dz.$$

(iii) Compute

$$\int_0^\infty \frac{1}{(3x^2 + 9)} dx$$

using contour integration.

Question 5 [15 marks]

A box contains the following 4 slips of paper, each having exactly the same dimensions:

(1) win prize 1; (2) win prize 2; (3) win prize 3; (4) win prize 1, 2 and 3.

One slip will be randomly selected with probability $\frac{1}{4}$. Let A_i be the event of winning prize i for $i = 1, 2, 3$. Show that A_1 and A_3 are independent but A_1, A_2 and A_3 are not mutually independent.

— **END OF PAPER** —