National University of Singapore
Faculty of Science
Department of Mathematics

Semester 1 Examination 2009-1010

QF4102 — Financial Modeling

Nov/Dec 2009 Time allowed: 2.5 hours.

INSTRUCTIONS TO CANDIDATES

1. Including this page, the examination paper comprises **Three (3)** printed pages.
2. This exam has **Four (4)** questions. Answer **All** questions.
3. This is a close-book examination. One page A4 size, double sides, handwritten help-sheet is permitted.
4. The marks for questions are not necessarily the same; marks for each question are indicated at the beginning of the question.

Question 1 [25 marks]

Consider an *American-style* lookback option with exercise payoff

$$S_t - m_t, \ t \in [0, T],$$

where S_t is the underlying asset price, $m_t = \min_{0 \le \tau \le t} S_\tau$, and T is the maturity.

(i) [5 marks] Write down the two-state variable binomial tree method for the pricing of the option.

(ii) [10 marks] Give the single-state variable binomial tree method for the pricing of the option, and plot a three-step single-state variable binomial tree.

(iii) [10 marks] Point out the shortage of the above single-state variable binomial tree method, and present an *improved* single-state variable binomial tree method.

Question 2 [20 marks]

Consider the following stochastic volatility model of a European call option:

$$\frac{\partial U}{\partial t} + \frac{1}{2} v S^2 \frac{\partial^2 U}{\partial S^2} + \rho c v S \frac{\partial^2 U}{\partial S \partial v} + \frac{1}{2} c^2 v \frac{\partial^2 U}{\partial v^2} + r S \frac{\partial U}{\partial S} + k(b - v) \frac{\partial U}{\partial v} - rU = 0$$

in $S > 0$, $v > 0$, $t \in [0, T)$, with the terminal condition

$$U(S, v, T) = (S - X)^+ .$$

Here, r, c, k, b, ρ, and X are all positive constants, and $\rho \in (0, 1)$. Give the fully implicit finite difference method for solving the above partial differential equation (the upwind scheme should be used, and the boundary conditions should be imposed).

Question 3 [25 marks]

Consider a European put option with an extra feature: the issuer (not the holder) has the right to buy it back with the predetermined price $M(t)$ at any time t during the option's life. That is, the option price has an upper bound $M(t)$.

(i) [7 marks] Write down the binomial tree method for the option pricing.

(ii) [8 marks] Assume that the underlying asset price follows a geometric Brownian motion. Write down the continuous-time partial differential equation (PDE) model for the option pricing.

(iii) [10 marks] Give a penalty method with Crank-Nicolson scheme for solving the PDE model in (ii), where the singularity of terminal condition should be removed.

Question 4 [30 marks]

Consider a *European-style* fixed strike *arithmetic* Asian call option with the terminal payoff

$$\left(\frac{1}{T}\int_0^T S_\tau d\tau - X\right)^+,$$

where X, T and S_t are the strike price, the maturity, and the underlying asset price at time t, respectively. Assume that the underlying asset price is governed by a jump-diffusion process:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dB_t - \frac{1}{2}dq_t,$$

where μ and σ are positive constants, B_t is a Brownian motion, and q_t is a Poisson process with constant intensity λ. Consider the Merton's jump diffusion model.

(i) [10 marks] Derive the partial differential equation model for pricing the Asian option.

(ii) [20 marks] Give the pseudo program code of the Monte-Carlo simulation for the option price and the hedging ratio (Delta), where the antithetic variance reduction technique should be used.

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