NATIONAL UNIVERSITY OF SINGAPORE

FACULTY OF SCIENCE

SEMESTER 1 EXAMINATION 2009-2010

MA2213 Numerical Analysis I

November 2009 — Time allowed : 2 hours

INSTRUCTIONS TO CANDIDATES

1. This is a closed book examination. Each student is allowed to bring one piece of A4-sized two-sided help sheet into the examination room.

2. This examination paper contains a total of **SIX (6)** questions and comprises **FOUR (4)** printed pages.

3. Answer **ALL** questions in **Section A**. Each question in Section A carries 20 marks.

4. Answer not more than **TWO (2)** questions from **Section B**. Each question in Section B carries 20 marks.

5. Candidates may use calculators. However, they should lay out systematically the various steps in the calculations.

SECTION A

Answer **all** *the questions in this section. Section A carries a total of 60 marks.*

Question 1. [20 marks]

(a) Given the equation $x^2 - 40x + 1 = 0$ and the quadratic formula

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

find the two roots (in decimal form) as accurately as possible using five-digit rounding arithmetic. (Do NOT use iterative methods.)

(b) Consider the linear system

$$\begin{aligned} 6x_1 + 2x_2 + 2x_3 &= -2 \\ 2x_1 + \frac{2}{3}x_2 + \frac{1}{3}x_3 &= 1 \\ x_1 + 2x_2 - x_3 &= 0. \end{aligned}$$

Solve the system by Gaussian elimination with partial pivoting and four-digit rounding arithmetic.

Question 2. [20 marks]

(a) The following data are taken from a polynomial of degree ≤ 5. What is the degree of the polynomial?

x	-2	-1	0	1	2	3
$p(x)$	-5	1	1	1	7	25

(b) Find the least squares polynomial approximation of degree one and degree two, respectively, for the function

$$f(x) = x^2 - 2x + 3$$

over the interval $[-1, 1]$.

Question 3. [20 marks]

(i) If $f(x)$ is a continuously differentiable function over the interval $[0, 1]$, then the length of the curve is given by

$$\int_0^1 (1 + |f'(x)|^2)^{1/2} dx.$$

Given

x	0	0.25	0.5	0.75	1
$f'(x)$	0	0.26	0.55	0.93	1.56

use Composite Trapezoidal rule with 4 subintervals to approximate the length of the graph of $f(x)$ over the interval $[0, 1]$. Then find an upper bound for the error in the approximation assuming $f(x) = -\ln(\cos x)$.

(ii) Use the table of data in part (i) and Romberg integration to obtain a better approximation for the length of the curve.

SECTION B

Answer not more than **two** *questions from this section. Section B carries a total of 40 marks.*

Question 4. [20 marks]

(a) How should c be chosen in order that the iteration

$$x_{n+1} = -c^2 + c^2 x_n + \sqrt{x_n}$$

converges to $\alpha = 1$ for any $x_0 \in [1, 2]$?

(b) In applying Newton's method for the root-finding problem $f(x) = 0$, we may not wish to compute the derivative at each step. Thus we could define an iteration formula by

$$x_{n+1} = z_{n+1} - \frac{f(z_{n+1})}{f'(x_n)}, \quad z_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

Suppose the sequence $\{x_n\}$ converges to the root α and $f(x)$ is three times differentiable and $f'(\alpha) \neq 0$. Show that the order of convergence of $\{x_n\}$ to α is at least 3.

Question 5. [20 marks]

(i) Gaussian quadrature on $[-1,1]$ with three node points is given by

$$G_3(f) = \frac{5}{9} f\left(\frac{-\sqrt{15}}{5}\right) + af(0) + \frac{5}{9} f\left(\frac{\sqrt{15}}{5}\right).$$

Determine the value of constant a and then use this quadrature formula to approximate

$$\int_0^1 \left[\ln x^2 + 2009(2x-1)^2 \tan(2x-1)\right] dx.$$

(ii) The three-point quadrature rule with error term is given by

$$\int_{-1}^1 f(x)dx = \frac{5}{9} f\left(\frac{-\sqrt{15}}{5}\right) + af(0) + \frac{5}{9} f\left(\frac{\sqrt{15}}{5}\right) + kf^{(6)}(\xi)$$

with the constant a found in part (i). Determine the constant k.

Question 6. [20 marks]

An experiment was conducted to measure the saturation values of dissolved oxygen concentration (mg/L) as a function of temperature (°C). The saturation values at different temperatures were measured to obtain the following table.

T	0	5	10	15
S_T	S_0	S_5	S_{10}	S_{15}

To approximate the saturation value at $T = 12$, an interpolation polynomial $P(x)$ of degree three was constructed. It gave an approximation $S_{12} \approx P(12) = 10$. The experimental data were then discarded.

However, it was discovered later that the value S_{10} was understated by 0.5 and the value S_{15} was overstated by 0.2 due to experimental mistakes.

If a new interpolation polynomial $H(x)$ is constructed using the data in the table with the revised values for S_{10} and S_{15}, what is the value of $H(12)$?

END OF PAPER