NATIONAL UNIVERSITY OF SINGAPORE

FACULTY OF SCIENCE

SEMESTER 1 EXAMINATION 2009-2010

MA1521 Calculus for Computing

November/December 2009 — Time allowed : 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains a total of **EIGHT (8)** questions and comprises **FOUR (4)** printed pages.
2. Answer **ALL** 8 questions. The marks for each question are indicated at the beginning of the question. The maximum score is **80 marks**.
3. **Write your matriculation number neatly on the front page of the answer booklet provided.**
4. **Write your solutions in the answer booklet. Begin your solution to each question on a new page.**
5. Calculators may be used. However, you should lay out systematically the various steps in your calculations.
6. **This is a CLOSED BOOK examination. One A4-sized helpsheet is allowed.**

Question 1 [10 marks]

(a) Let

$$f(x) = \begin{cases} \dfrac{x^3+3x^2-16x+12}{x^2-3x+2} & \text{if } x \neq 1,\ x \neq 2 \\ \\ 8 & \text{if } x = 1 \text{ or } x = 2 \end{cases}$$

Determine whether f is continuous at $x = 1$ and $x = 2$. Justify your answer.

(b) Let

$$g(x) = \frac{\ln x}{x^2}, \quad \text{where} \quad x > 0.$$

(i) Find the largest open interval on which g is increasing.

(ii) Find the largest open interval on which the graph of g is concave up.

Question 2 [10 marks]

(a) A long straight metal beam of length L metres initially rested with its top end against a vertical wall of a building and its bottom end on the horizontal ground. The beam then begins slipping. When the top end is 12 metres above the ground, it is slipping down the wall at 8 metres per second. At this instance, the bottom end is slipping across the ground at 6 metres per second. Find the exact value of L.

(b) The region R in the first quadrant is bounded by the curve

$$y = \frac{2}{\sqrt{x^2-6x+8}},$$

the $x-$axis, and the lines $x = 6$ and $x = 8$. Find the volume of the solid generated when R is revolved about the $x-$axis.

(*Give the exact volume in terms of* π.)

Question 3 [10 marks]

Determine whether each of the following series converges or diverges. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n+3}{n(n+1)}$.

(b) $\displaystyle\sum_{n=2}^{\infty}\frac{(\ln n)^2}{n^2+8}$.

Question 4 [10 marks]

(a) Find the interval of convergence of the following power series:

$$1 \;-\; \frac{1}{2}\,(x-6) \;+\; \frac{1}{4}\,(x-6)^2 \;-\; + \;\cdots\; + \;\left(-\frac{x-6}{2}\right)^n \;+\; \cdots$$

On the interval of convergence, find the sum of the power series.

(b) Use the Maclaurin series of $\dfrac{1}{1-x}$ to find the exact value of $\displaystyle\sum_{n=1}^{\infty}\frac{n^2}{5^n}$.

Question 5 [10 marks]

(a) The plane Π passes through the point $P_0(1,8,1)$ and is perpendicular to the two planes Π_1 and Π_2, where

$\Pi_1 : \quad x+y-2z \;=\; 2, \qquad\qquad \Pi_2 : \quad x+3y-4z \;=\; 2.$

Find a Cartesian equation of Π.

(b) Let

$$f(x,y) \;=\; x^2 - xy + 3y^2.$$

At the point $P(2,1)$, find a direction, given by a unit vector $\mathbf{u}$, such that the directional derivative $D_{\mathbf{u}}f(P)$ is <u>*zero*</u>.

Question 6 [10 marks]

(a) Find the local maximum, local minimum and saddle points (if any) of

$$f(x, y) \;=\; 2x^3 + 6xy + y^2.$$

(b) Use Lagrange multipliers to find the point (points) on the sphere

$$x^2 + y^2 + z^2 = 189$$

at which the function $g(x, y, z) = x + 2y + 4z$ has a maximum value.

Question 7 [10 marks]

(a) Using a suitable substitution, solve the differential equation

$$\frac{dy}{dx} \;=\; \frac{x - 2y}{3x - 6y + 4}\;.$$

(b) Solve the differential equation

$$\frac{dy}{dx} \;-\; y \;=\; \cos x \;-\; \sin x.$$

Question 8 [10 marks]

Solve the following differential equations:

(a) $2y\dfrac{dy}{dx} \;-\; 1 \;=\; 4xy^2 + 4x + y^2.$

(b) $\dfrac{d^2y}{dx^2} \;-\; 9y \;=\; (12x + 8)e^{3x}.$

END OF PAPER