NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF MATHEMATICS

SEMESTER 2 EXAMINATION 2008-2009

QF4102 — Financial Modelling

May 2009 Time allowed: 2.5 hours.

INSTRUCTIONS TO CANDIDATES

1. Including this page, the examination paper comprises **Three (3)** printed pages.
2. This exam has **Four (4)** questions. Answer **All** questions.

Question 1 [20 marks]

Consider a European-style lookback option with payoff

$$\max_{0 \le t \le T} S_t - S_T,$$

where S_t is the underlying asset price, T the maturity and X the strike price. Assume that S_t follows a geometric Brownian motion.

(i) [10 marks] Write the single-state variable binomial tree method and the corresponding partial differential equation (PDE) model.

(ii) [10 marks] How do we discretize the Neuman boundary condition in the PDE model such that the resulting explicit difference scheme is better than the binomial tree method?

Question 2 [30 marks]

Assume that in a jump-diffusion market the risk neutral process of the stock price follows

$$\frac{dS_t}{S_t} = \tilde{r}dt + \sigma dB_t + (Y - 1)dq_t,$$

where B_t is a standard Brownian motion, $\tilde{r}$, σ, and Y are constants, and q_t is a Poisson process with intensity λ. There is no correlation between the Brownian motion and the Poisson process. Consider a European vanilla call option written on the stock in the market.

(i) [20 marks] Give the pseudo-code of the Monte Carlo simulation for the option price and the Delta, where the antithetic variance reduction should be used.

(ii) [10 marks] Give the partial differential equation model and an efficient finite difference scheme.

Question 3 [20 marks]

Using a three-step binomial tree model, describe how to replicate a European-style Asian option with payoff

$$\left(\frac{1}{T}\int_0^T S_t dt - X\right)^+ ,$$

here S_t is the underlying asset price, T the maturity and X the strike price.

Question 4 [30 marks]

Consider the following pricing model of a European-style option on two assets S_1 and S_2:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma_1^2 S_1^2 \frac{\partial^2 V}{\partial S_1^2} + \rho\sigma_1\sigma_2 S_1 S_2 \frac{\partial^2 V}{\partial S_1 \partial S_2} + \frac{1}{2}\sigma_2^2 S_2^2 \frac{\partial^2 V}{\partial S_2^2} + rS_1 \frac{\partial V}{\partial S_1} + rS_2 \frac{\partial V}{\partial S_2} - rV = 0$$

in $S_1 > 0$, $S_2 > 0$, $t \in [0, T)$, with the terminal condition

$$V(S_1, S_2, T) = (S_1 - S_2 - X)^+ .$$

Here, r, σ_1, σ_2, ρ and X are all positive constants, and $\rho \in (0, 1)$.

(i) [10 marks] Give the fully implicit finite difference scheme (the boundary conditions should be prescribed).

(ii) [10 marks] Write the corresponding American-style option pricing model and present a penalty method with the Crank-Nicolson scheme.

(iii) [10 marks] Assume that short selling of the stock S_1 is not permitted. Give the definition of the utility indifference bid price (from the point of view of the option's buyer) of the European-style option, where the exponential utility function $U(x) = -e^{-\alpha x}$, $\alpha > 0$ is used.

– **End of Paper** –