NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF MATHEMATICS

SEMESTER 2 EXAMINATION 2008-2009

QF3101 Investment Instruments: Theory and Computation

April/May 2009 — Time allowed : 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains a total of **FIVE (5)** questions and **ONE** (1) appendix and comprises **SIX (6)** printed pages (including this cover page).
2. Answer **ALL** questions. The mark for each question is indicated at the beginning of the question.
3. Start your answer to each question on a new page.
4. This is a closed book examination. Use of help sheets is **not** allowed.
5. You may use a calculator. However, you should lay out systematically the various steps in the calculations.
6. Express all numerical answers up to **4 decimal places**.

Question 1 [*20 marks*]

Let R_i denote the excess return over the risk-free rate for stock i, that is, $R_i = r_i - r_f$. Likewise, the market portfolio's excess return is $R_M = r_M - r_f$. Suppose that the single factor model for stocks 1 and 2 is estimated with the following results:

$$\begin{aligned} R_1 &= 0.8\% + 0.55R_M + e_1 \\ R_2 &= -1.5\% + 1.35R_M + e_2 \\ \sigma_M &= 23\%, \quad \sigma(e_1) = 15\%, \quad \sigma(e_2) = 8\%. \end{aligned}$$

(i) Recall that the variance for r_i is defined as $\sigma_i^2 = \mathrm{E}[(r_i - \overline{r}_i)^2]$. Use this definition to show that

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma_{e_i}^2, \quad i = 1, 2$$

where β_i is the factor loading for stock i in the given one-factor model.

(ii) Determine the standard deviation of the rate of return for both stocks.

(iii) Determine the correlation between the rates of return of the two stocks.

(iv) If the use of one-factor APT is appropriate, and the expected rates of return for stocks 1 and 2 are 15.25% and 19.75% respectively, determine the risk free rate λ_0 and the factor price λ_1 for this APT model.

Question 2 [*20 marks*]

(a) The current stock price of the bank Rock Solid is \$5 per share. It is known that Rock Solid will pay a dividend of \$0.15 per share six months from now. Suppose the current 6-month and 1-year spot rates are 2.2% and 2.7% (annualized, semi-annual compounding) respectively, and a forward contract is to be initiated today on 100,000 shares of Rock Solid for delivery in 1 year.

(i) Determine the forward price for this forward contract.

(ii) Suppose six months later, immediately after the dividend payment, the stock price has risen to \$8 per share, and the 6-month spot rate is 3.2% (annualized, semi-annual compounding). Determine the value of the long position on this contract at that time.

(b) A US investor will receive a sum of money in one year's time. She wants to invest the sum in a one-year US Treasury note, and thus approaches a bank to act as the counter party in a forward contract to purchase the Treasury note. Given that the interest rates for 1-year, 1.5-year and 2-year maturities are 1.2%, 1.3% and 1.8% (annualized, semi-annual compounding) respectively, determine the forward price (per \$100 face value) for this forward contract if the Treasury note has a coupon rate of 5%.

Question 3 [*20 marks*]

(a) Consider an interest rate swap configured on yearly in-arrear exchanges of a fixed rate payment for a floating level payment linked to 1-year LIBOR on a notional principal of \$100,000. This swap was initiated one year ago, and still has two more years before maturity. The first exchange has just taken place, and the swap is currently valued at $-\$6,572.96$ for the fixed rate payer, with the current 1-year and 2-year LIBOR being 3.2% and 4.3% (annualized, yearly compounding) respectively.

  (i) Determine the swap rate of the given interest rate swap.

  (ii) If the swap rate for a newly initiated three-year interest rate swap (fixed versus 1-year LIBOR) is 5.4159%, determine the implied 3-year LIBOR (annualized, yearly compounding).

(b) Company A, a Malaysian manufacturer, wishes to borrow Singapore dollars at a fixed rate of interest. Company B, a Singapore multinational, wishes to borrow Malaysian Ringgit at a fixed rate of interest. They have been quoted the following rates per annum:

	Singapore Dollars	**Malaysian Ringgit**
Company A	6.6%	7.2%
Company B	7.0%	8.0%

Design a currency swap, with a bank acting as intermediary, that will produce a saving of 15 basis points per annum for each of the two companies.

Question 4 [*20 marks*]

(a) Suppose a Treasury bill (TB1) with 68 days to maturity is being quoted today at a dollar price of \$98.23, and another Treasury bill (TB2) with 158 days to maturity is being quoted today at a dollar price of \$97.14.

  (i) Determine the discount yield and investment rate for Treasury bill TB1.

  (ii) Determine the futures price (index basis) of a Treasury bill futures which expires 68 days from now.

(b) Suppose today is 16 March 2009, and the following table gives a summary of information on five Eurodollar futures with settlement prices recorded at the end of the day.

Contract	Settlement Price	Contract Expiry Date	#Days from 16-March to Expiry Date
Mar 09	98.645	16-Mar-2009	0
Jun 09	98.785	14-Jun-2009	90
Sep 09	98.805	13-Sep-2009	181
Dec 09	98.665	13-Dec-2009	272
Mar 10	98.605	15-Mar-2010	364

(i) Determine the implied 6-month LIBOR.

(ii) Determine the fair fixed rate for a 6x12 forward rate agreement initiated today.

Question 5 [*20 marks*]

(a) An investor in Singapore has a portfolio of three positions: 1-year USD zero coupon bond (position 1), a position in German stock index DAX (position 2) and a SGD/USD foreign exchange position (position 3). The 1-day 95% SGD VaRs for positions 1, 2 and 3 are 3,410, 21,920 and 10,580 respectively. The correlation matrix is given as follows.

Position	1-Year USD	DAX	SGD/USD
1-Year USD	1	0.09	−0.55
DAX	0.09	1	−0.78
SGD/USD	−0.55	−0.78	1

(i) Compute the 1-day 95% SGD VaR of this portfolio.

(ii) Use your answer to (i) to obtain the 10-day 99% SGD VaR.

(b) A bond trading book holds $1 million zero coupon bond due to mature in 2.25 years which falls between the standard maturities of 2 and 3 years. To correctly capture the volatility of this position in the bank's VaR estimate, cash flow mapping is used. The following information is available:

Standard Maturity	Yield (%)	Price Volatility (daily %)	Correlation Matrix, R	
			2 years	3 years
2 years	6.28	0.25	1	0.95
3 years	6.39	0.32		1

Determine the 1-day 95% VaR of this zero-coupon bond position.

END OF PAPER

Appendix: QF3101 Formula Sheet (Two pages)

For asset i, we have

- Capital Asset Pricing Model (CAPM): $\overline{r}_i - r_f = \beta_i(\overline{r}_M - r_f)$, where $\beta_i = \sigma_{iM}/\sigma_M^2$.
- Systematic and Firm-Specific Risk: $\sigma_i{}^2 = \beta_i^2\sigma_M^2 + \text{var}(\epsilon_i)$

Single Factor Models For asset i, we have $r_i = a_i + b_i f + e_i, i = 1, 2, \ldots, n$, and

$$
\begin{aligned}
\bar{r}_i &= a_i + b_i\bar{f}, & \sigma_i^2 &= b_i^2\sigma_f^2 + \sigma_{e_i}^2\\
\sigma_{ij} &= b_i b_j \sigma_f^2, \qquad i \neq j, & b_i &= \text{cov}(r_i, f)/\sigma_f^2\\
R^2 &= (\text{var}(r_i) - \text{var}(e_i))/\text{var}(r_i). & &
\end{aligned}
$$

For a portfolio of assets following single-factor models: $\sigma_p^2 = b^2\sigma_f^2 + \sigma_e^2$ where $b = \sum_{i=1}^n w_i b_i$, $e = \sum_{i=1}^n w_i e_i$ and $\sigma_e^2 = \sum_{i=1}^n w_i^2\sigma_{e_i}^2$.

Two-Factor Models For asset i, we have $r_i = a_i + b_{1i}f_1 + b_{2i}f_2 + e_i, i = 1, 2, \ldots, n$, and

$$
\begin{aligned}
\overline{r}_i &= a_i + b_{1i}\overline{f}_1 + b_{2i}\overline{f}_2,\\
\text{cov}(r_i, r_j) &= b_{1i}b_{1j}\text{var}(f_1) + (b_{1i}b_{2j} + b_{2i}b_{1j})\text{cov}(f_1, f_2) + b_{2i}b_{2j}\text{var}(f_2) + \text{cov}(e_i, e_j)\\
\text{cov}(r_i, f_1) &= b_{1i}\,\text{var}(f_1) + b_{2i}\,\text{cov}(f_1, f_2)\\
\text{cov}(r_i, f_2) &= b_{1i}\,\text{cov}(f_1, f_2) + b_{2i}\,\text{var}(f_2).
\end{aligned}
$$

Arbitrage Pricing Theory For asset i on m-factor model: $r_i = a_i + \sum_{j=1}^m b_{ji}f_j + e_i$, APT implies that
$\overline{r}_i = \lambda_0 + \sum_{j=1}^m b_{ji}\lambda_j$, where λ_0 is the risk-free rate, and λ_j is the factor price for factor j, $j = 1, \ldots, n$.

Forward Price Formulae

- $F_0 = S_0/d(0,T)$,
- $F_0 = (S_0 - I)/d(0,T)$,
- $F_0 = S_0 e^{(r-q)T}$
- $F_0 = \dfrac{S_0}{d(0,M)} + \displaystyle\sum_{k=0}^{M-1}\dfrac{c(k)}{d(k,M)}$
- $F_0 = \dfrac{S_0}{d(0,M)} + \displaystyle\sum_{k=0}^{M-1}\dfrac{c(k)}{d(k,M)} - \sum_{k=0}^{M-1}\dfrac{y}{d(k,M)}$

Interest Rate Swap The initial value of an interest rate swap with maturity T to the company receiving floating and paying fixed on notional principal N is

$$V = \left(1 - d(0,T) - r\sum_{i=1}^{M} d(0,t_i)\right)N.$$

The value of swap with time t_1 to the next exchange for the company receiving fixed and paying floating is

$$V = Nd(0,T) + \sum_{i=1}^{n} k\ d(0,t_i) - (N + f_1)d(0,t_1).$$

Currency Swap The value of swap for the company paying currency 1 and receiving currency 2 is $V = SP_2 - P_1$, where S is the spot exchange rate, and P_i, $i = 1, 2$, are the values of the bonds in currencies 1 and 2 respectively.

Hedge Basis $b_i = S_i - F_i$, Effective price for hedging $F_1 + b_2$,
Minimum variance hedge ratio $\beta^* = \dfrac{\mathrm{cov}(S_T, F_T)}{\mathrm{var}(F_T)}$.

Changing beta of a Portfolio No. of index contracts= $(\beta_1 - \beta_2)\dfrac{S}{m\, I_0}$.

T-bill discount yield= $\dfrac{100 - P}{100} \times \dfrac{360}{t}$, Investment rate (less than half year) $i = \dfrac{100 - P}{P} \times \dfrac{y}{t}$.

For more than one half-year to maturity, solve $(t/(2y) - 0.25)i^2 + (t/y)i + (P - 100)/P = 0$.

Full price of a bond= $\displaystyle\sum_{i=0}^{n-1} \frac{C}{\left(1 + \frac{r}{m}\right)^{i+w}} + \frac{F}{\left(1 + \frac{r}{m}\right)^{w+n-1}}$, accrued interest= $(1 - w)C$.

Add-on yield = $\dfrac{\text{Interest}}{\text{Principal}} \times \dfrac{360}{\text{Days to maturity}}$.

Eurodollar Futures Price Index Price= 100−(annualized add-on yield for the 3-month period).

FRA payoff to buyer $(y - X) \times N \times \dfrac{m}{360} \times \dfrac{1}{1 + y(m/360)}$.

Forward rate for $[T, T + m]$ $f_L(T, T + m) = \left(\dfrac{d_L(T)}{d_L(T + m)} - 1\right) \Big/ (m/360)$.

PV of FRA $[f_L(T, T + m) - X] \times (m/360) \times d_L(T + m)$.

Currency Forward Forward Price $F_0 = x_0\, d(r_f, T)/d(r_l, T)$, r_f and r_l are foreign and local interest rates respectively.

Value-at-Risk
For single position, relative VaR: $\mathrm{VaR}_r = \alpha\sigma W_0$, absolute VaR: $\mathrm{VaR}_a = (\alpha\sigma - \mu)W_0$.

$\alpha = 1.65$ for 95% confidence level, and $\alpha = 2.33$ for 99% confidence level.

Square root of time rule N-day VaR= 1-day VaR$\times\sqrt{N}$.

Delta-Normal method Portfolio VaR is $\mathrm{VaR}_p = \alpha\sqrt{\mathbf{x}'\mathbf{\Sigma}\mathbf{x}}$.

Cash Flow Mapping For mapping zero at t to zeros at standard maturities p and q with $p < t < q$.
Solve quadratic for alpha: $(\sigma_p^2 + \sigma_q^2 - 2\rho_{p,q}\sigma_p\sigma_q)\alpha^2 + (2\rho_{p,q}\sigma_p\sigma_q - 2\sigma_q^2)\alpha + (\sigma_q^2 - \sigma_t^2) = 0$.