NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF MATHEMATICS

SEMESTER EXAMINATION FOR THE DEGREE OF B.SC.

SEMESTER 1 EXAMINATION 2008–2009

MA2213 NUMERICAL ANALYSIS I

November 2008– Time allowed : 2 hours

Instructions to Candidates

1. This examination paper contains a total of **Five (5)** questions and comprises **Three (3)** printed pages.
2. Answer **ALL** questions.
3. Candidates may use calculators. However, they should lay out systematically the various steps in the calculations.
4. All questions carry equal marks.

Question 1 [20 marks]

Solve the following linear system of equations by Gaussian elimination with partial pivoting with four-digit arithmetic with rounding

$$\begin{aligned} 0.003000x_1 + 59.14x_2 &= 59.17 \\ 5.291x_1 - 6.130x_2 &= 46.78 \\ 0.600x_3 &= 1.00 \\ 0.002110x_4 + 0.08204x_5 &= 0.04313 \\ 0.3370x_4 + 12.84x_5 &= 6.757. \end{aligned}$$

Question 2 [20 marks]

(a) Find the degree of the minimal degree polynomial interpolating the following data

x	-2	-1	0	1	2	3
$f(x)$	2	8	22	32	26	-8

(b) Let m and n be two positive integers with $n \geq m$. Let $f(x) = x^m$. Assume x_0, x_1, $\cdots$, and x_n are $n+1$ real numbers. Calculate the divided difference

$$f[x_0, x_1, \cdots, x_n].$$

Question 3 [20 marks]

(a) Let $T(a,b)$ and $T(a, \frac{a+b}{2}) + T(\frac{a+b}{2}, b)$ be the single and the composite Trapezoidal rule with $n = 2$ to $\int_a^b f(x)dx$. Derive the relationship between

$$\|T(a,b) - T(a, \frac{a+b}{2}) - T(\frac{a+b}{2}, b)\|$$

and

$$\| \int_a^b f(x)dx - T(a, \frac{a+b}{2}) - T(\frac{a+b}{2}, b)\|.$$

(b) Find the degree of precision of the following formula

$$I(f) = \frac{1}{90}[7f(0) + 32f(\frac{1}{4}) + 12f(\frac{1}{2}) + 32f(\frac{3}{4}) + 7f(1)]$$

to approximate $\int_0^1 f(x)dx$.

Question 4 [20 marks]

Assume that the formula

$$I(f) = \sum_{i=0}^{6} A_i f(x_i)$$

approximating $\int_{-1}^{1} f(x)dx$ is exact for all polynomials of degree at most 6 and the distinct nodes x_i $(i = 0, 1, \cdots, 6)$ are symmetrically placed about the origin, compute the error

$$E(x^7) = \int_{-1}^{1} x^7 dx - \sum_{i=0}^{6} A_i x_i^7.$$

Question 5 [20 marks]

The polynomial $q(x) = x^4 - x^3 + x^2 - x + 1$ assumes the following values:

x	-2	-1	0	1	2	3
$q(x)$	31	5	1	1	11	61

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Find a polynomial $p(x)$ with degree at most 5 that takes these values:

x	-2	-1	0	1	2	3
$p(x)$	31	5	1	1	11	30

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— END OF PAPER —