Partial Solutions to MA3501 2008 Paper

The following are remarks to the solutions of Questions 2 and 3

2(a). It follows from Tutorial 3 Question 1 that

$$u(x,t) = \frac{1}{2} + \sum_{n=1}^{\infty} \left[(-1)^n - 1\right] \frac{2}{n^2\pi^2} \cos{(n\pi x)} e^{-n^2\pi^2 t}.$$

(b). Note that $(-1)^n - 1 = 0$ when n is even and that $(-1)^n - 1 = -2$ when n is odd.

3. The solution comes from Chapter 2, pages 65-67.

Note that $\chi_{(-at,at)}(\tau) = 1$ if τ lies in $(-at, at)$ and

$\chi_{(-at,at)}(\tau) = 0$ if τ does not lie in $(-at, at)$.

Exam 2008

Q1 (a)

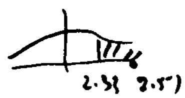
$H_0: \mu = 8390$

$H_1: \mu \neq 8350$

$$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \simeq \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{8275-8350}{\frac{1500}{\sqrt{900}}} = -2.24$$

Reject Ho, i.e., we have enough evidence to conclude that the mean cost of raising a child is significantly diff. from \$8350 at 5% level of significance.

(b) $H_0: \mu = 30 \quad (\mu \leq 30)$



$H_1: \mu > 30$

$$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \simeq \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{31.5-30}{3.5/\sqrt{36}} = 2.57$$

Reject Ho, accept H_1

Q 2 (a) See tutorial 3, Q1

(b)

$$u(x,0) = x = \frac{1}{2} - \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos(2n-1)\pi x$$

Q 3 $\quad u(x,t) = \frac{1}{2a}\int_{-\infty}^{\infty} f(x-\tau)\,\chi_{(-at,at)}(\tau)\,d\tau$

$$u(x,t) = \frac{1}{2a}\int_{-at}^{at} f(x-\tau)\,d\tau$$

$$u(\pi, \tfrac{\pi}{2}) = \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x-\tau)\,d\tau$$

$$= \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(x-\tau)\,d\tau = -1$$

Q4 $\quad x'(t) = e^t \Rightarrow x(t) = e^t + x_0$

$\frac{d}{dt}u(x(t),t) = x(t) \Rightarrow u(x(t),t) = e^t + x_0 t + C$

$\neq xt + C$

(see tut 5, Q3)

1.

Q5

$v''(x) = 0$

$v'(x) = 2[v(0)-2]$

$v(2) = -3$

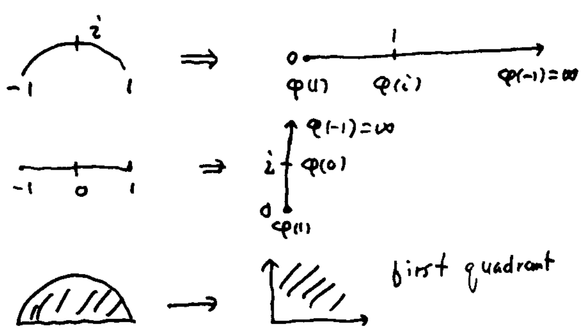
$\Rightarrow v(x) = Ax + B \Rightarrow v'(x) = A = 2[v(0)-2]$

$= 2[B-2]$

$\Rightarrow v(x) = -2x + 1 \quad v(2) = 2A + B = -3$

Q6

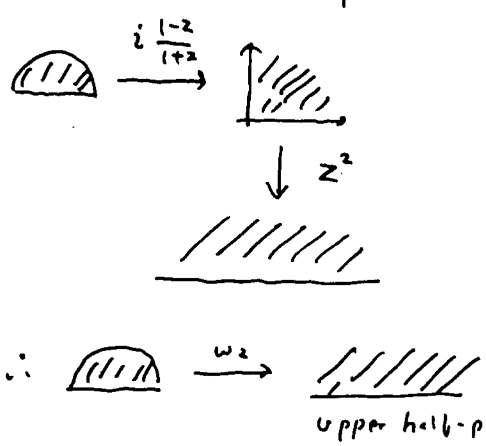
(b) (i)



(b) (ii)

$$\omega_2 = -\left(\frac{1-z}{1+z}\right)^2 = \left(i\,\frac{1-z}{1+z}\right)^2$$

$$= \omega_1^2$$



Q7

(a) Arg w is the imaginary part of

$\mathrm{Log}\, w = \ln|w| + i\,\mathrm{Arg}\, w$

Arg $(w-1)$ is the imaginary part of

$\mathrm{Log}(w-1) = \ln|w-1| + i\,\mathrm{Arg}(w-1)$

$\log w$ and $\mathrm{Log}(w-1)$ are analytic in the upper-half plane

$\therefore$ Arg w and Arg $(w-1)$ are soln of Laplace eq on the upper-half plane

$\therefore$ $a\,\mathrm{Arg}\, w - b\,\mathrm{Arg}(w-1)$ is a soln of Laplace eq in the upper-half plane.

Note that on the boundary (x-axis)

$w = u + vi = u$

$\Phi(u) = a\pi$ if $0 < u < 1$ $\left(\begin{matrix}\mathrm{Arg}\, w = \pi \\ \mathrm{Arg}(w-1) = 0\end{matrix}\right)$

$= (a-b)\pi$ if $u < 0$ ($\mathrm{Arg}\, w = \mathrm{Arg}(w-1) = \pi$)

$= 0$ if $u > 1$

$(\mathrm{Arg}\, w = \mathrm{Arg}(w-1) = 0)$

(b) when $a = b = \frac{100}{\pi}$

$\Phi(u) = 100$ if $0 < u < 1$

$= 0$ if $u > 1$, $u < 0$

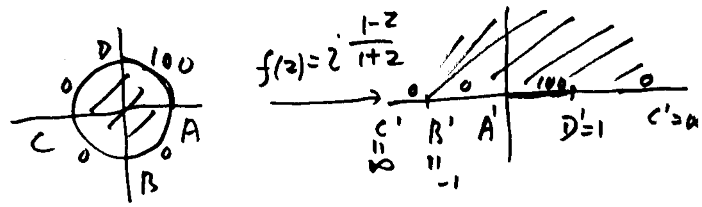
$\Phi(w) = \frac{100}{\pi}\,\mathrm{Arg}(w-1) - \frac{100}{\pi}\,\mathrm{Arg}\, w$

is a soln of Laplace eq with

$\Phi(w) = 100$ if $0 < w < 1$

$\Phi(w) = 0$ if $w > 0$, $w > 1$

where $w = u + iv = u$



$\phi(z) = \Phi(f(z))$

$= \frac{100}{\pi}\,\mathrm{Arg}\left(i\,\frac{1-z}{1+z} - 1\right) - \frac{100}{\pi}\,\mathrm{Arg}\left(i\,\frac{1-z}{1+z}\right)$

is a soln of Laplace eq with

$\phi(z) = 100$ on $\overset{\frown}{AD}$

$= 0$ on $\overset{\frown}{BCD}$