Ph.D. Qualifying Examination

Complex Analysis

Sem 1, 2006/2007

1. (a) Find the number of zeroes, counting multiplicities, of the polynomial

$$f(z) = 2z^5 - 6z^2 - z + 1 = 0$$

in the annulus $1 \leq |z| \leq 2$.

(b) Evaluate the improper integral

$$\int_0^\infty \frac{x^2 dx}{(x^2+1)(x^2+4)}.$$

2. Suppose that $p(z) = a_0 + a_1 z + \cdots + a_n z^n$ is a polynomial such that $|p(z)| \leq 1$ for all $|z| = 1$. Show that $|a_k| \leq 1$ for $k = 0, 1, \ldots, n$. Justify your answer carefully; state clearly any results used.

3. Suppose that f and g are entire functions such that $|f(z)| \leq |g(z)|$ for all $z \in \mathbb{C}$. Show that there is a constant c such that $g(z) = cf(z)$ for all $z \in \mathbb{C}$. Justify your answer carefully.

4. Find an analytic isomorphism from the open region between $|z| = 1$ and $|z - \frac{1}{2}| = \frac{1}{2}$ to the upper half plane $\text{Im} z > 0$. (You may leave your result as a composition of functions).

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