

## Ph.D. Qualifying Examination

Complex Analysis

Sem 1, 2005/2006

1. State and prove Liouville's Theorem and use it to prove the Fundamental Theorem of Algebra.

2. Find the number of roots of

$$z^7 - 5z^3 + 9 = 0$$

that lie between the circles  $|z| = 1$  and  $|z| = 2$ . Justify your answer carefully.

3. Use contour integration to evaluate the integral

$$\int_0^{\infty} \frac{1}{x^6 + 1}.$$

4. Find a conformal map that maps the region  $D_1 = \{z \in \mathbb{C} : 0 < \text{Im } z < 1\}$  onto the open unit disk  $D_2 = \{z \in \mathbb{C} : |z - 3| < 1\}$ .

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