**Ph.D. Qualifying Examination**

Algebra

Semester 2, 2004/2005

## Ring Theory

(a) Prove that every integral domain can be imbedded in a field.

(b) Let $D$ be an integral domain and let $F = \{x \in D : xd = 1 \text{ for some } d \in D\}$. Suppose that $D$ is a finite dimensional vector space over $F$. Prove that $D$ is a field.

## Group Theory

(a) Let $G$ be a nonabelian finite group generated $x$ and $y$, where $o(x) = o(y) = 2$. Prove that $G$ is isomorphic to a dihedral group.

(b) Let $G$ be a group of order 56. Suppose that $G$ has 2 or more subgroups of order 7. Prove that $G$ has a subgroup isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$.

## Field Theory

(a) Let $F$ be a finite field. Prove that $F - \{0\}$ under multiplication is a cyclic group.

(b) Prove that $\sqrt{2} + \sqrt{3} + \sqrt{5} + \sqrt{7}$ is irrational.

END OF PAPER