Ph.D. Qualifying Examination : Algebra

Ring Theory

(a) Let p be a prime. Find all the rings (up to ring isomorphism) of p elements. Justify your answers.
(b) Let $R = M_n(\mathbb{R})$ be the ring of all $n \times n$ matrices over the real numbers. Find all the ideals of R. Justify your answers.

Group Theory

(a) Let p be a prime. Find all the groups (up to group isomorphism) of order $2p$. Justify your answers.
(b) Let p be a prime and let G be a group of p^3 elements. Suppose that G is not abelian. Prove that $Z(G)$ is cyclic of order p.

Field Theory

(a) Let p be a prime. Find all the fields (up to field isomorphism) of p^2 elements Justify your answers.
(b) Let σ be a field automorphism from $\mathbb{R}$ to $\mathbb{R}$. Prove that $\sigma(x) = x$ for all $x \in \mathbb{R}$.