**Ph.D. Qualifying Examination**
**Sem 1, 2002/2003**
**Linear Algebra**

1.(a) Let $(x-1)^4(x-2)^3$ and $(x-1)^2(x-2)$ be the characteristic and minimal polynomials of $A$ respectively. Determine the Jordan canonical forms of $A$.

(b) The order (if exists) of a matrix $A$ is the smallest positive integer $n$ such that $A^n = I$. Find the smallest positive integer $m$ such that $M_m(\mathbb{Q})$ (the set of all $m \times m$ matrices with rational entries) admits a matrix of order $n$.

2. Let $A$ and $B$ be two $n \times n$ matrices. Suppose that $AB = BA$. Determine whether the following hold. Justify your answer.

(a) There exists a rational basis $\mathcal{B}$ (a basis with rational entries) such that both $[A]_{\mathcal{B}}$ and $[B]_{\mathcal{B}}$ are upper triangular.

(b) There exists a complex basis $\mathcal{B}$ (a basis with complex entries) such that both $[A]_{\mathcal{B}}$ and $[B]_{\mathcal{B}}$ are upper triangular.

3. Let $A$ be a matrix. Prove that the column rank of $A$ equals the row rank of $A$.

**— END OF PAPER —**