Ph.D. Qualifying Examination
Sem 2, 2000/2001
Complex Analysis

1. Suppose that both $f(z)$ and $\overline{f(z)}$ are analytic in a domain D. Prove that $f(z)$ is constant in D.

2. Let
$$f(z) = \begin{cases} \dfrac{e^z - 1}{z} & \text{if } z \neq 0 \\ 1 & \text{if } z = 0. \end{cases}$$

   (i) Prove that f is entire.

   (ii) Find $f^{(99)}(0)$.

3. Let
$$f(z) = \frac{z(z-2)^2}{\cos \pi z - 1}.$$

   (i) Identify the singular points of f and classify each as a removable singularity, an essential singularity or a pole of specific order.

   (ii) Evaluate
$$\int_\gamma f(z)\, dz$$
   where γ is the positively oriented circle $|z| = 1$.

4. Use the Cauchy residue theorem to evaluate the integral
$$\int_{-\infty}^{\infty} \frac{x^2}{x^4 + 1}\, dx.$$

5. Find a conformal map that maps the region $D_1 = \{z \in \mathbb{C} : \ |z| > 1\}$ onto the region $D_2 = \{z \in \mathbb{C} : \ \mathrm{Re}(z) < 0\}$.

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