2007/2008 SEMESTER 1 MID-TERM TEST

MA1505 MATHEMATICS I

October 2, 2007

8:30pm to 9:30pm

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY:

1. This test paper consists of **TEN (10)** multiple choice questions and comprises **Eleven (11)** printed pages.
2. Answer all 10 questions. 1 mark for each correct answer. No penalty for wrong answers. Full mark is 10.
3. All answers (Choices A, B, C, D, E) are to be submitted using the pink form (FORM CC1).
4. Use **only 2B pencils** for FORM CC1.
5. On FORM CC1 (section B), **write** your **matriculation number** and **shade** the corresponding numbered circles **completely**. Your FORM CC1 will be graded by a computer and it will record a **ZERO** for your score if your matriculation number is not correct.
6. Write your full name in section A of FORM CC1.
7. Only circles for answers 1 to 10 are to be shaded.
8. For each answer, the circle corresponding to your choice should be **properly** and **completely** shaded. If you change your answer later, you must make sure that the original answer is properly erased.
9. For each answer, **do not shade more than one circle**. The answer for a question with more than one circle shaded will be marked wrong.
10. **Do not fold** FORM CC1.
11. Submit FORM CC1 before you leave the test hall.

1. Let $f(x)$ be a differentiable function which satisfies $f(5) = 3$ and $f'(5) = 2$. Find the value of the expression $\frac{d}{dx}\left[x^2 f(x)\right]$ at the point $x = 5$.

(A) 80

(B) 88

(C) 90

(D) 98

(E) 100

2. A leaky water tank is in the shape of an inverted right circular cone with depth 5 m and top radius 2 m. At a time when the water in the tank is 4 m deep, it is leaking out at a rate of $\frac{1}{10}$ m^3/min. How fast is the water level in the tank dropping at that time?

(A) $\frac{25}{64\pi}$ m/min.

(B) $\frac{5}{64\pi}$ m/min.

(C) $\frac{2}{5\pi}$ m/min.

(D) $\frac{4}{25\pi}$ m/min.

(E) $\frac{5}{128\pi}$ m/min.

3. A lighthouse L is located on a small island 5 km north of a point A on a straight east-west shoreline. A cable is to be laid from L to a point B on the shoreline 10 km east of A. The cable will be laid through the water in a straight line from L to a point C on the shoreline between A and B, and then from C to B along the shoreline. The part of the cable lying in the water costs \$5000 per km, and the part along the shoreline costs \$4000 per km. Find the minimum total cost of the cable.

(A) \$55000

(B) \$55500

(C) \$54000

(D) \$53500

(E) \$52500

4. If $y^3 + xy - 1 = 0$, then $y' =$

(A) $\frac{1}{3y}$

(B) $-\frac{x}{x+3y^2}$

(C) $\frac{x}{x+3y^2}$

(D) $-\frac{y}{x+3y^2}$

(E) $\frac{y}{x+3y^2}$

5. Let $f(x)$ be a differentiable function with $f'(x)$ continuous. If

$$\lim_{x\to 3}\frac{f(x)-2\sqrt{x+1}}{x^2-9}=-\frac{1}{16},$$

then $(f(3))(f'(3)) =$

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) $\frac{3}{16}$

(E) -16

6. A curve on the xy-plane passes through the points $(1, 5)$ and $\left(a^2, b\right)$ where a is a positive constant. If the slope at each point (x, y) on the curve is $\frac{3}{\sqrt{x}}$, then $b =$

(A) $2a + 3$

(B) $3a + 2$

(C) $6a - 1$

(D) $6 - a$

(E) $8 - 3a$

7. Let n be a positive odd integer. Then $\int_0^{\frac{\pi}{2}} \cos x \sqrt{\sin^n x}\, dx =$

(A) $\frac{1}{n+2}$

(B) $\frac{2}{n}$

(C) $\frac{1}{n}$

(D) $\frac{1}{n+1}$

(E) $\frac{2}{n+2}$

8. $\frac{d}{dx}\int_0^{x^2}\sqrt{2-\sin^3 t}\,\mathrm{d}t =$

(A) $2x\sqrt{2-\sin^3 x^2}$

(B) $2x\sqrt{2-\sin^3 x}$

(C) $\frac{2x}{\sqrt{2-\sin^3 x^2}}$

(D) $\frac{2x}{\sqrt{2-\sin^3 x}}$

(E) $\frac{-3\sin^2 x}{2\sqrt{2-\sin^3 x^2}}$

9. Let a be a positive constant. Find the area of the bounded plane region between the parabola $y^2 = x + 12a^2$ and the straight line $y = \frac{1}{a}x$.

(A) $\frac{397}{6}a^3$

(B) $\frac{665}{6}a^3$

(C) $\frac{343}{6}a^3$

(D) $\frac{301}{6}a^3$

(E) $\frac{73}{6}a^3$

10. A solid of revolution is generated by rotating the finite plane region bounded by the curve $y = x^2$ and the line $y = 1$ about the line $y = 1$. Find its volume.

(A) $\frac{14\pi}{13}$

(B) $\frac{16\pi}{15}$

(C) $\frac{6\pi}{5}$

(D) $\frac{8\pi}{7}$

(E) $\frac{15\pi}{14}$

END OF PAPER

National University of Singapore
Department of Mathematics

2007-2008 Semester 1   MA1505 Mathematics I   Mid-Term Test Answers

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Question	1	2	3	4	5	6	7	8	9	10
Answer	A	E	A	D	B	C	E	A	C	B

MA1505 2007 Test Solutions

1). $\frac{d}{dx}\{x^2 f(x)\} = 2x\, f(x) + x^2 f'(x)$

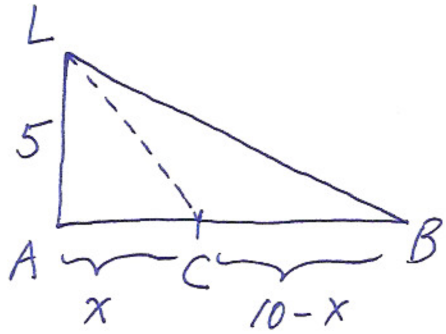
at $x=5$, $\frac{d}{dx}\{x^2 f(x)\} = 2(5)f(5) + 5^2 f'(5) = 30 + 50 = \underline{\underline{80}}$ (A)

2). $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{2}{5}h\right)^2 h = \frac{4\pi}{75}h^3$

$$\frac{dV}{dt} = \frac{4\pi}{25}h^2\frac{dh}{dt}$$

at $h=4$, $\frac{dV}{dt} = \frac{1}{10} \Rightarrow \frac{1}{10} = \frac{4\pi}{25}(4)^2\frac{dh}{dt} \Rightarrow \underline{\underline{\frac{dh}{dt} = \frac{5}{128\pi}}}$ (E)

3). $LC = \sqrt{5^2 + x^2}$



Cost $y = 5000\sqrt{25+x^2} + 4000(10-x)$

$$\frac{dy}{dx} = \frac{5000x}{\sqrt{25+x^2}} - 4000$$

$$\frac{dy}{dx} = 0 \Rightarrow 5x = 4\sqrt{25+x^2}$$

$$\Rightarrow 25x^2 = 16(25+x^2) \Rightarrow x = \frac{20}{3}$$

$$\Rightarrow y = 5000\sqrt{25^2+\left(\frac{20}{3}\right)^2} + 4000\left(\frac{10}{3}\right) = \underline{\underline{55000}}$$ (A)

4). $y^3 + xy - 1 = 0$

$3y^2y' + xy' + y = 0 \Rightarrow y' = \dfrac{-y}{x+3y^2}$ (D)

5). $\because \lim_{x\to 3}(x^2-9) = 0$

$\therefore \lim_{x\to 3}\{f(x) - 2\sqrt{x+1}\} = 0$ (If not, then $\lim_{x\to 3}\dfrac{f(x)-2\sqrt{x+1}}{x^2-9}$ will not exist)

$\therefore f(3) = 4$

$\therefore \lim_{x\to 3}\dfrac{f(x)-2\sqrt{x+1}}{x^2-9} = \lim_{x\to 3}\dfrac{f'(x) - \frac{1}{\sqrt{x+1}}}{2x} = \dfrac{f'(3)-\frac{1}{2}}{6}$

$\therefore \dfrac{f'(3)-\frac{1}{2}}{6} = -\dfrac{1}{16} \Rightarrow f'(3) = \dfrac{1}{8}$

$\therefore (f(3))(f'(3)) = (4)(\frac{1}{8}) = \dfrac{1}{2}$ (B)

6). $y = \int \frac{3}{\sqrt{x}}\,dx = 6\sqrt{x} + C$

$x=1,\ y=5 \Rightarrow 5 = 6 + C \Rightarrow C = -1$

$\therefore y = 6\sqrt{x} - 1$

$\therefore x = a^2 \Rightarrow y = 6\sqrt{a^2} - 1$

$= 6a - 1$ ($\because a > 0$)

$\therefore b = 6a - 1$ (C)

7). $\int_0^{\pi/2} \cos x \sin^{n/2} x \, dx = \int_0^{\pi/2} \sin^{n/2} x \, d(\sin x)$

$= \frac{1}{n/2+1} \sin^{(n/2+1)} x \Big|_0^{\pi/2} = \frac{2}{n+2}$ (E)

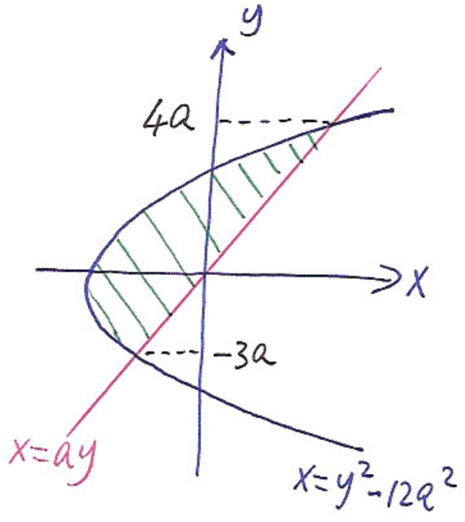
8). $\frac{d}{dx}\int_0^{x^2} \sqrt{2-\sin^3 t}\, dt = 2x\sqrt{2-\sin^3 x^2}$ (A)

9). $x = y^2 - 12a^2$ and $x = ay$

$\Rightarrow y^2 - ay - 12a^2 = 0$

$\Rightarrow (y-4a)(y+3a) = 0$

$\Rightarrow y = 4a, \; y = -3a$



Area $= \int_{-3a}^{4a} \{ay - (y^2 - 12a^2)\}\, dy$

$= \frac{343}{6} a^3$ (C)

10). Volume $= 2\int_0^1 \pi(1-x^2)^2 dx$

$= \frac{16}{15}\pi$ (B)