**2006/2007 SEMESTER 1 MID-TERM TEST**

**MA1505 MATHEMATICS I**

**October 2, 2006**

**SESSION 2 : 7:30 - 8:30pm**

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**PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY:**

1. This test paper consists of **TWELVE (12)** multiple choice questions and comprises **Seven (7)** printed pages.
2. Answer all 12 questions. 1 mark for each correct answer. No penalty for wrong answers. Full mark is 12.
3. All answers (Choices A, B, C, D, E) are to be submitted using the pink form (FORM CC1).
4. Use **only 2B pencils** for FORM CC1.
5. On FORM CC1 (section B), **write** your **matriculation number** and **shade** the corresponding numbered circles carefully. Your FORM CC1 will be graded by a computer and it will record a **ZERO** for your score if your matriculation number is not correct.
6. Write your full name in section A of FORM CC1.
7. Only circles for answers 1 to 12 are to be shaded.
8. For each answer, the circle corresponding to your choice should be properly shaded. If you change your answer later, you must make sure that the original answer is properly erased.
9. For each answer, **do not shade more than one circle**. The answer for a question with more than one circle shaded will be marked wrong.
10. **Do not fold** FORM CC1.
11. Submit FORM CC1 before you leave the test hall.

1. Let $f\left(x\right)=\ln\frac{\cos 3x}{\sin 2x}$, where $0<x<\frac{\pi}{6}$. Then $f'\left(x\right)\;=$

**(A)** $-\left(3\tan 3x+2\cot 2x\right)$

**(B)** $-\left(3\tan 3x-2\cot 2x\right)$

(**C**) $3\tan 3x+2\cot 2x$

(**D**) $-\left(3\cot 3x+2\tan 2x\right)$

(**E**) $3\cot 3x-2\tan 2x$

2. The equation of a curve C is $x^3+xy+2y^3=0$. The tangent line to C at the point $(-1,1)$ intersects the $y$-axis at the point $(0,k)$. Find the value of $k$.

**(A)** $\frac{2}{5}$

**(B)** $-\frac{4}{5}$

**(C)** $\frac{1}{4}$

**(D)** $\frac{1}{5}$

**(E)** $\frac{4}{5}$

3. A ladder 25 feet long is leaning against a vertical wall. The bottom of the ladder starts to slide away at a constant rate of 6 feet/minute. How fast is the top of the ladder moving down the wall when it is 24 feet above the ground?

**(A)** $\frac{7}{24}$ feet/minute

**(B)** $\frac{7}{4}$ feet/minute

**(C)** 6 feet/minute

**(D)** $\frac{1}{4}$ feet/minute

**(E)** $\frac{7}{25}$ feet/minute

4. Evaluate $\lim_{x\to 0} \frac{1-\left(1-x^2\right)^n}{1-\cos 3x}$, where $n$ is a positive constant.

**(A)** $n$

**(B)** $\frac{2n}{3}$

**(C)** $\frac{n}{3}$

**(D)** $\frac{2n}{9}$

**(E)** $\frac{n}{9}$

5. Evaluate $\int_n^{n+1} (n+x)^2 (n-x)^{10}\, dx$ , where $n$ is a constant.

**(A)** $\frac{4n^2}{11} - \frac{n}{3} - \frac{1}{13}$

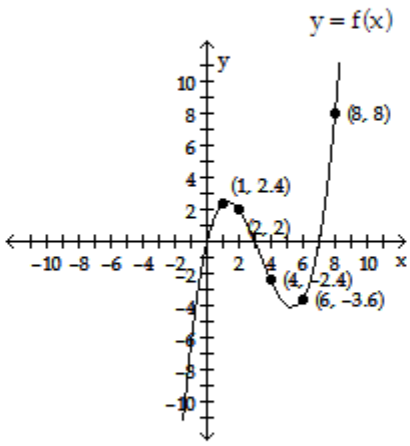
**(B)** $\frac{4n^2}{11} - \frac{n}{3} + \frac{1}{13}$

**(C)** $\frac{4n^2}{11} + \frac{n}{3} + \frac{1}{13}$

**(D)** $\frac{4n^2}{11} + \frac{n}{3} - \frac{1}{13}$

**(E)** None of the above

6. Let $f(x)$ be a differentiable function whose graph is shown in the figure. The position, measured from the origin in meters, at time t seconds, of a particle moving along the $x$-axis is given by the formula $s = \int_0^t f(x)\, dx$. What is the particle's velocity at $t = 6$ seconds?




**(A)** 3.6 m/sec

**(B)** 0 m/sec

**(C)** -3.6 m/sec

**(D)** 1.8 m/sec

**(E)** None of the above

7. Let $R$ be the region in the first quadrant bounded above by the line $y = \sqrt{2}$, below by the curve $y = \sec 3x$, on the left by the $y$-axis and on the right by the point of intersection of the line $y = \sqrt{2}$ and the curve $y = \sec 3x$. Find the volume of the solid generated by revolving $R$ about the line $y = 0$.

**(A)** $\frac{1}{3}\pi^2 - \frac{1}{6}\pi$

**(B)** $\frac{1}{6}\pi^2 + \frac{1}{3}\pi$

**(C)** $\frac{1}{3}\pi^2 + \frac{1}{6}\pi$

**(D)** $\frac{1}{6}\pi^2 - \frac{1}{3}\pi$

**(E)** $\frac{1}{4}\pi + \frac{1}{3}$

8. $\int_{-1}^{6} \left|4x - x^2\right| dx =$

**(A)** $\frac{71}{3}$

**(B)** $\frac{128}{3}$

**(C)** $\frac{64}{3}$

**(D)** $64$

**(E)** $\frac{113}{3}$

9. $\int_0^{\pi/4} \frac{1}{\cos^2 x+3\sin^2 x} dx =$

**(A)** $\frac{\pi}{3}$

**(B)** $\frac{\sqrt{3}\pi}{5}$

**(C)** $\frac{\sqrt{3}\pi}{9}$

**(D)** $\frac{\pi}{9}$

**(E)** $\frac{\pi}{4}$

10. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{4^n (2n)!}{(n!)^2} (x+1)^n$.

**(A)** $\infty$

**(B)** $\frac{1}{4}$

**(C)** 16

**(D)** 4

**(E)** $\frac{1}{16}$

11. Find the Taylor series of $f(x) = \frac{1}{(2x-1)^3}$ at $a = 1$.

**(A)** $\sum_{n=0}^{\infty} (-1)^{n+1} (n-1) 2^n (x-1)^n$

**(B)** $\sum_{n=0}^{\infty} (-1)^n \frac{n+2}{2^{n+1}} (x-1)^n$

**(C)** $\sum_{n=0}^{\infty} (-1)^n (n+2) 2^{n-1} (x-1)^n$

**(D)** $\sum_{n=0}^{\infty} (-1)^n (n+1)(n+2) 2^{n-1} (x-1)^n$

**(E)** $\sum_{n=0}^{\infty} (-1)^n n(n+1) 2^{n-1} (x-1)^n$

12. Let $\frac{d}{dx}\left(\frac{x^{10}}{1-x}\right) = \sum_{n=0}^{\infty} a_n x^n$. Then $a_{2006} =$

**(A)** 1996

**(B)** 2016

**(C)** 2005

**(D)** 2006

**(E)** 2007

END OF PAPER

# National University of Singapore
## Department of Mathematics

2006-2007 Semester 1 MA1505 Mathematics I Mid-Term Test Session 2 Answers

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| Question | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Answer | A | D | B | D | C | C | D | A | C | E | D | E |

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# Session 2 Hints and Solutions

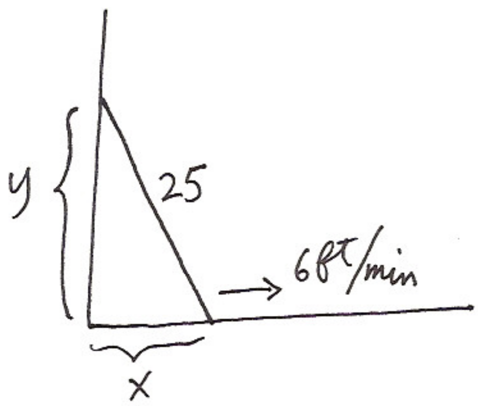
1). $f(x) = \ln \frac{\cos 3x}{\sin 2x} = \ln \cos 3x - \ln \sin 2x, \cdots$

2). $3x^2 + y + xy' + 6y^2y' = 0.$

at $(-1, 1)$, $y' = -\frac{4}{5}$

$\therefore \frac{k-1}{0-(-1)} = -\frac{4}{5} \Rightarrow k = \frac{1}{5}$

3).




$x^2 + y^2 = 25^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$6x + y\frac{dy}{dt} = 0 \quad (\because \frac{dx}{dt} = 6)$

$\therefore \frac{dy}{dt} = -\frac{6x}{y}$

$y = 24 \Rightarrow x = \sqrt{25^2 - 24^2} = 7 \Rightarrow \frac{dy}{dt} = -\frac{7}{4}$

$\therefore$ The ladder is moving down the wall at 7/4 ft/min

4). $\lim_{x \to 0} \frac{1-(1-x^2)^n}{1-\cos 3x} = \lim_{x \to 0} \frac{n(1-x^2)^{n-1}(2x)}{3\sin 3x}$

$= \left\{\lim_{x \to 0} \frac{n(1-x^2)^{n-1}}{3}\right\}\left\{\lim_{x \to 0} \frac{2x}{\sin 3x}\right\}$

$= \left(\frac{n}{3}\right)\left(\frac{2}{3}\right) = \frac{2n}{9}$

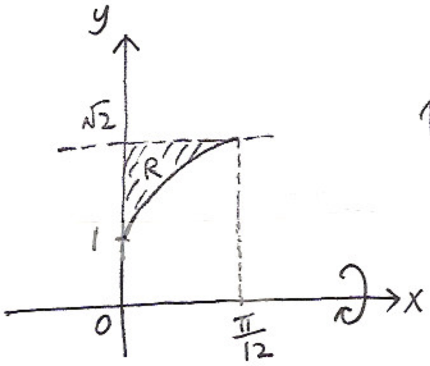
5). $\int_n^{n+1}(n+x)^2(n-x)^{10}\,dx = \int_0^{-1}(2n-u)^2u^{10}(-du)$ (let $u=n-x$)

$= \int_{-1}^{0}(4n^2u^{10}-4nu^{11}+u^{12})\,du$

⋮

6). $\frac{ds}{dt} = f(t) \Rightarrow$ at $t=6$, $\frac{ds}{dt} = f(6) = \underline{\underline{-3.6}}$

7) Solve $\sqrt{2} = \sec 3x \Rightarrow$ first point of intersection in the first quadrant is $x = \frac{\pi}{12}$




$$\text{Volume} = \int_0^{\frac{\pi}{12}} \pi\{(\sqrt{2})^2-(\sec 3x)^2\}\,dx$$

$$= \pi\left[2x-\frac{1}{3}\tan 3x\right]_0^{\frac{\pi}{12}} = \underline{\underline{\pi\left(\frac{\pi}{6}-\frac{1}{3}\right)}}$$

8). $4x-x^2 = x(4-x)$

$-\quad 0 \quad +\quad 4 \quad -$

$$\int_{-1}^{6}|4x-x^2|\,dx = -\int_{-1}^{0}(4x-x^2)\,dx+\int_0^4(4x-x^2)\,dx-\int_4^6(4x-x^2)\,dx$$

⋮

$= \underline{\underline{\frac{71}{3}}}$

9). $$\int_0^{\pi/4}\frac{1}{\cos^2x+3\sin^2x}\,dx = \int_0^{\pi/4}\frac{\sec^2x}{1+3\tan^2x}\,dx$$

$$= \frac{1}{\sqrt{3}}\int_0^{\pi/4}\frac{d(\sqrt{3}\tan x)}{1+(\sqrt{3}\tan x)^2}$$

$$= \frac{1}{\sqrt{3}}\left[\tan^{-1}(\sqrt{3}\tan x)\right]_0^{\pi/4} = \frac{1}{\sqrt{3}}\left(\frac{\pi}{3}\right) = \underline{\underline{\frac{\sqrt{3}\pi}{9}}}$$

10). $$\left| \frac{\dfrac{4^{n+1}(2n+2)!}{[(n+1)!]^2}(x+1)^{n+1}}{\dfrac{4^n(2n)!}{(n!)^2}(x+1)^n} \right| = \frac{4(2n+2)(2n+1)}{(n+1)^2}|x+1|$$

$$\longrightarrow 16|x+1|$$

$$16|x+1| < 1 \Longleftrightarrow |x-(-1)| < \frac{1}{16}$$

11). Differentiate $\frac{1}{1-r} = \sum_{n=0}^{\infty} r^n$ with respect to $r$ two times:

$$\therefore \frac{1}{(1-r)^2} = \sum_{n=1}^{\infty} n r^{n-1}$$

$$\therefore \frac{2}{(1-r)^3} = \sum_{n=2}^{\infty} n(n-1)r^{n-2} = \sum_{n=0}^{\infty} (n+2)(n+1)r^n$$

$$\therefore \frac{1}{(1-r)^3} = \sum_{n=0}^{\infty} \frac{1}{2}(n+2)(n+1)r^n.$$

$$f(x) = \frac{1}{(2x-1)^3} = \frac{1}{[2(x-1)+1]^3} = \frac{1}{\{1-[-2(x-1)]\}^3}$$

$$= \sum_{n=0}^{\infty} \frac{1}{2}(n+2)(n+1)(-1)^n 2^n (x-1)^n = \sum_{n=0}^{\infty} (-1)^n (n+2)(n+1) 2^{n-1} (x-1)^n$$

12). $$\frac{d}{dx}\left(\frac{x^{10}}{1-x}\right) = \frac{d}{dx}\left\{x^{10}\sum_{n=0}^{\infty} x^n\right\} = \frac{d}{dx}\sum_{n=0}^{\infty} x^{n+10}$$

$$= \sum_{n=0}^{\infty} (n+10)x^{n+9}$$

Put $n = 1997 \Longrightarrow a_{2006} = 1997 + 10 = 2007$