Matriculation Number:

NATIONAL UNIVERSITY OF SINGAPORE

MA1505 - MATHEMATICS 1

AY2013/2014 : Semester 2

Time allowed : 2 hours

INSTRUCTIONS TO CANDIDATES

1. Write your matriculation number neatly in the space above. Do not write your name.
2. Do not insert loose papers into this booklet. This booklet will be collected at the end of the examination.
3. This examination paper contains **EIGHT (8)** questions and comprises **THIRTY-THREE (33)** printed pages.
4. Answer **ALL** 8 questions. The marks for each question are indicated at the beginning of the question. The maximum possible score for this examination is **80 marks**.
5. Write your solution in the space below each question.
6. This is a CLOSED BOOK examination. One A4-sized helpsheet is allowed.
7. Programmable calculators and graphing calculators are disallowed. Scientific calculators may be used. However, you should lay out systematically the various steps in your calculations.

For official use only. Do not write in the boxes below.

Question	1	2	3	4	5	6	7	8
(a)								
(b)								

Question 1 (a) [5 marks]

Given

$$f(x, y) = x^{35}(y + 1)e^{42y},$$

find the value of the product $\dfrac{\partial f}{\partial x} \cdot \dfrac{\partial f}{\partial y}$ when $x = 1$ and $y = 0$.

(*More space for the solution to Question 1 (a).*)

Question 1 (b) [5 marks]

Let

$$f(x, y) = xy^5.$$

If $Q(q, 0)$ is a point on the x-axis such that the *directional derivative* of $f(x, y)$ at $P(1, 1)$ in the direction given by $\overrightarrow{PQ}$ is *zero*, find the value of q.

(*More space for the solution to Question 1 (b).*)

Question 2 (a) [5 marks]

Find the local maximum, local minimum and saddle points, if any, of the function

$$f(x, y) = 3x^2 + y^3 - 12xy.$$

(*More space for the solution to Question 2* (*a*).)

Question 2 (b) [5 marks]

In a model of a company producing three products X, Y and Z with respective quantities x, y and z, suppose that manufacturing constraints force

$$2x^2 + y^2 + 4z^2 \;=\; 8,800.$$

If the profit $P(x, y, z)$ of the company can be modelled by

$$P(x, y, z) \;=\; x + 2y + 2z,$$

use the Lagrange multiplier method to find the maximum profit.

(*More space for the solution to Question 2 (b).*)

Question 3 (a) [5 marks] (Multiple Choice Question)

Let

$$f(x) = \begin{cases} 0 & \text{if } 0 < x < \pi \\ \frac{\pi}{2}\cos x & \text{if } \pi < x < 2\pi \end{cases}$$

Suppose that the sine half range expansion of $f(x)$ is

$$f_s(x) = \sum_{n=1}^{\infty} b_n \sin\frac{nx}{2}.$$

Find the exact value of b_{3012} .

Options:

(A) $-\dfrac{1505}{1506 \cdot 1507}$

(B) $-\dfrac{1506}{1505 \cdot 1507}$

(C) $-\dfrac{1507}{1505 \cdot 1506}$

(D) $\dfrac{1505}{1506 \cdot 1507}$

(E) None of the above

State your option/answer: ________________

(*More space for the solution to Question 3* (*a*).)

Question 3 (b) [5 marks]

Let $f(x) = x^2 + x$ for $-\pi < x < \pi$, and $f(x + 2\pi) = f(x)$ for all x. Suppose that the Fourier series of $f(x)$ is

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx).$$

Find the exact value of $\sum_{n=1}^{\infty} b_n$.

(*Hint*: $\ln(1 + x) = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n}$ for $-1 < x \leq 1$. *You need not prove this.*)

(*More space for the solution to Question 3* (*b*).)

Question 4 (a) [5 marks]

Find the exact value of the iterated integral

$$\int_0^{2014} \int_0^{8} \frac{x^{2013}}{x^{2014} + 8^{2014}} \, dx \, dy.$$

(*More space for the solution to Question 4 (a).*)

Question 4 (b) [5 marks]

The solid region D in the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$) is bounded by the three planes $z = 0$, $y = 0$, $y = \sqrt{3}\,x$ and the ellipsoid

$$x^2 + y^2 + 4z^2 = 1.$$

Find the exact volume of D.

(*More space for the solution to Question 4 (b).*)

Question 5 (a) [5 marks]

The surface S is the portion of the paraboloid

$$z = x^2 + y^2$$

below the plane $z = 12$. Find the exact area of S.

(*More space for the solution to Question 5 (a).*)

Question 5 (b) [5 marks]

Find the exact value of the iterated integral

$$\int_0^1 \int_{-\cos^{-1} y}^{\cos^{-1} y} e^{2\sin x}\, dx\, dy,$$

where $0 \leq \cos^{-1} y \leq \pi$.

(*More space for the solution to Question 5 (b).*)

Question 6 (a) [5 marks]

Let C be the curve in the xy-plane given by

$$C \ : \quad \mathbf{r}(t) \ = \ (4\cos t)\,\mathbf{i} \ + \ (4\sin t)\,\mathbf{j}, \quad \text{where} \ \ 0 \leq t \leq \pi.$$

If

$$\mathbf{F}(x, y) \ = \ y^2\,\mathbf{i} \ - \ xy\,\mathbf{j},$$

find the value of the line integral $\displaystyle\int_C \mathbf{F} \bullet d\mathbf{r}$.

(*More space for the solution to Question 6 (a).*)

Question 6 (b) [5 marks]

The curve C in the xy-plane is given by

$$C \ : \quad \mathbf{r}(t) \ = \ e^{t(\pi-2t)}\Big[\,(\pi\cos t)\,\mathbf{i} \ + \ (t\sin t)\,\mathbf{j}\,\Big], \quad 0 \le t \le \frac{\pi}{2}\,.$$

Find the exact value of the line integral

$$\int_C (1 - e^x\cos y)\ dx \ + \ e^x \sin y\, dy.$$

(*More space for the solution to Question 6 (b).*)

Question 7 (a) [5 marks]

The region R in the xy-plane is triangular with vertices $O(0,0)$, $A(12,0)$ and $B(7,2)$. An object M at the point $P(x,y)$ is moved by a force $\mathbf{F}$ given by

$$\mathbf{F}(x,y) \;=\; \left[\cos\left(e^x\right) + 2x^3y^2 + 3y\right]\mathbf{i} \;+\; \left[e^{\sin y} + x^4y + 7x\right]\mathbf{j}.$$

Use Green's Theorem to find the work done $\oint_C \mathbf{F} \bullet d\mathbf{r}$ when M is moved once along the (closed) boundary C of R in the anticlockwise direction.

(*More space for the solution to Question 7 (a).*)

Question 7 (b) [5 marks]

The surface S is the portion in the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$) of the sphere of radius 4 and centred at the origin. Find the exact value of the surface integral

$$\iint_S x\, dS.$$

(*More space for the solution to Question 7 (b).*)

Question 8 (a) [5 marks]

The surface S is given by

$$S \ : \quad z \ = \ 9 - x^2 - 9y^2, \quad z \geq 0.$$

If

$$\mathbf{F}(x, y, z) \ = \ \left(x^7 e^x - 6y\right) \mathbf{i} \ + \ y^8 \mathbf{j} \ + \ z e^{xy} \mathbf{k},$$

find the exact value of the surface integral $\displaystyle\iint_S \text{curl } \mathbf{F} \bullet d\mathbf{S}$, where the orientation of S is given by the outer normal vector.

(*Hint: area of ellipse with equation* $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} \ = \ 1$ *is* πab*. You need not prove this.*)

(*More space for the solution to Question 8* (*a*).)

Question 8 (b) [5 marks]

The tetrahedron D is a solid region in the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$) bounded by the three coordinate planes $x = 0$, $y = 0$, $z = 0$ and the plane $x + y + z = 1$. Let S be the surface of D and $\mathbf{F}$ be the vector field

$$\mathbf{F}(x, y, z) \;=\; \left(x^2 + z^2\right)\mathbf{i} \;+\; xy\mathbf{j} \;+\; x^3y^3\mathbf{k}.$$

Given that $\iiint_D (x+1)\,dV \;=\; \dfrac{5}{24}$, find the exact value of the surface integral $\iint_S \mathbf{F} \bullet d\mathbf{S}$, where the orientation of S is given by the outer normal vector.

(*More space for the solution to Question 8* (*b*).)

END OF PAPER